A Dynamic Programming Approach to the Design of Composite Aircraft Wings[1]

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Abstract. A light and reliable aircraft is a major goal of aircraft designers. Since a wing skin comprises more than fifty percent of the wing's weight, it should be designed as efficiently as possible. The wing skin consists of many different types of material and thickness configurations at various locations. Selecting a thickness for each location is a significant design task. In this paper we formulate discrete models to determine optimal thicknesses for three different criteria: maximum reliability, minimum weight, and a tradeoff between the two. Since these formulations represent generalized discrete resource-allocation problems for which dynamic programming is well suited, we use this method to solve them. To reduce computations, a dynamic programming decomposition is applied.

Key Words. Wing design, maximum reliability, minimum weight, Pareto optimality, dynamic programming**.**

1. Introduction

This paper presents a dynamic programming approach for optimizing aircraft wing design whose outline, both in platform and cross-sectional shape, is robust enough to house a structure capable of

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doing its job. After aerodynamic analysis has determined a basic wing shape, this preliminary design should be modified to yield the lightest structure with sufficient strength and stiffness having a minimum of manufacturing problems (Ref. 1). A wing box is composed of skins (upper skin, lower skin, and substructure), spars, and ribs. Upper and lower skins play different roles in a wing box. The upper skin is loaded primarily in compression; therefore its design has to prevent buckling. The lower skin is loaded primarily in tension, so its material lay-up defined in Ref. 2 should ensure high tensile strength.

Wing skins of composite material consist of many different lay-ups and thickness configurations at locations determined typically by stress analysis. Such a location encounters high stress due to lift and drag forces on an aircraft wing. We discretize the region around the aerofoil using these analysis locations. There is no established procedure for breaking up the structure into panels for probabilistic analysis. In general, a probabilistic model panel within a structure is chosen to represent an area such that the internal stress and material strength are approximately constant over that area. There could be several hundred panels in a probabilistic analysis model, and a baseline thickness must be selected for each panel.

In general, the wing skins account for fifty to seventy percent of a wing's structural weight. If the thickness is increased at a location, the weight of that location will increase. If the location thickness is decreased while the applied load is kept constant, the internal stress will increase and the reliability of that location will decrease. The choice of thickness for each location is essential for an optimum wing box design balancing weight and reliability requirements.

In this paper we develop mathematical programming models to solve the following problems: (1) minimization of wing weight for a given reliability by selecting a thickness for each panel, (2) maximization of wing reliability for a given weight by selecting a thickness for each panel, and (3) investigation of tradeoffs between reliability and weight.

Recent related work includes that of Luo (Ref. 3), Pettit (Ref. 4), and Padmanabhan (Ref. 5), who apply optimization methods to generate designs that are both more reliable and more robust to uncertainties than the conventional designs. Pettit further presents a framework for wing design integrating structural and load analysis, reliability analysis, and optimization with most-probable point estimation (Ref. 6). Sobieszczanski-Sobieski uses an optimization approach for the design of an aircraft wing structural box with hundreds of design variables and thousands of constraints (Ref. 7).

Here we first formulate models for solving the above problems. Second, we present a solution approach that combines ideas from finite-element modeling, dynamic programming, and multi-objective programming. Third, we discuss the models. Finally, we offer future directions for research.

2. Formulation of the optimization problem

To illustrate our approach, relationships among reliability, thickness, and weight will be developed from Lear Fan 2100 Jet data provided to Northrop Grumman Commercial Aircraft Division (NGCAD) by the Federal Aviation Administration (FAA). We first note that weight is a product of area, thickness, and density. For each wing location, weight increases linearly as thickness is increased since the area and density of a location are constant. Baseline thickness is the standard thickness determined by deterministic structural analysis for finding the actual thickness at a particular location of the wing. Actual thickness is the actual measurement of the thickness at a particular location. For example, the baseline thickness of the first location for the Lear Fan 2100 is 0.2 inches. Therefore, the actual thickness corresponding to the ratio 0.95 is 0.2 times 0.95, or 0.19 inches.

Figure 1 represents a typical thick box beam wing structure, containing three spars and numerous ribs. This wing box structure usually serves as a fuel tank as well. A wing is divided into three major components upper skin, lower skin and substructure. The wing model for the upper skin is illustrated in Figure 2.

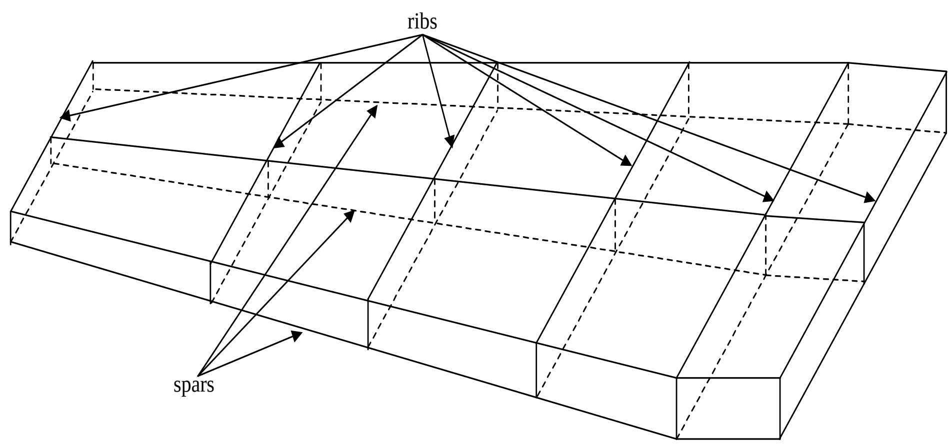

Figure 1. Typical wing structure

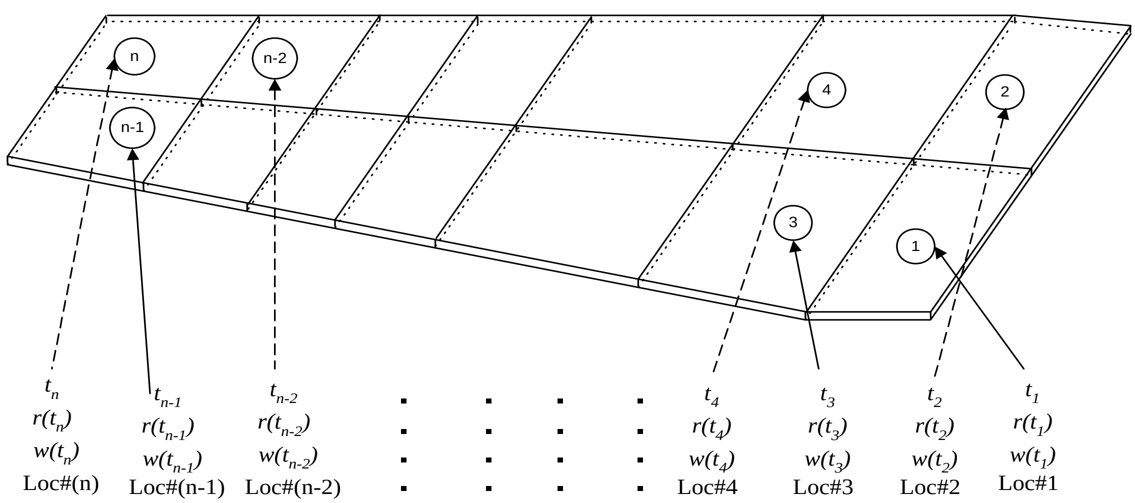

Figure 2. Configuration of a wing skin

Next we assume that internal stress decreases with an increase in thickness. It follows that reliability increases with a decrease in internal stress, which implies an increase in thickness and weight. In particular, wing reliability can approach 1.0 when the thickness of a wing panel is extremely thick and heavy. However, reliability grows nonlinearly with increasing thickness as depicted in Figure 3 for panel 1 of the Lear Fan 2100 Jet. The reasons for this nonlinear relationship are that the reliability is determined from the joint probability of two nonlinear functions and that the failure mode of this location is buckling, which is a nonlinear function of thickness.

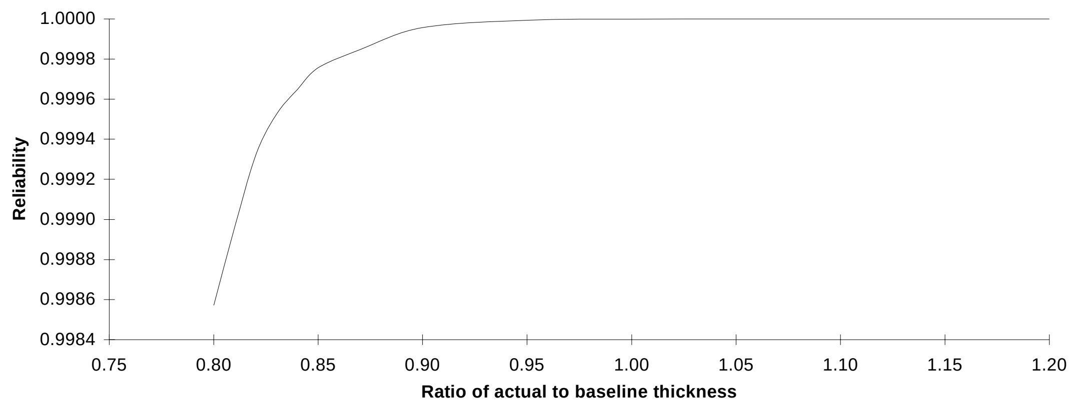

Figure 3. Relationship between reliability and thickness

Now for panel i = 1,2,3,...,n, let t_i denote its thickness, with a resulting reliability $r(t_i)$ and weight $w(t_i)$. Then for a sufficiently large n determined by the deterministic structural analysis the total reliability and weight for the upper skin can be estimated by the expressions

$$\text{total weight} = \sum_{i=1}^{n} w(t_i) \qquad (1)$$

$$\text{total reliability} = \prod_{i=1}^{n} r(t_i), \qquad (2)$$

where t_i is associated with the panel *i*. Assuming the t_i are assumed to be independent of each other, we arrive at following problem formulations.

2.1 Weight minimization problem (W): Our first objective is to minimize the total weight of a wing within a specified minimum reliability level $0 < r_0 < 1$ by selecting a thickness for each wing panel. From equations (1) and (2), the weight problem (W) can be formulated as the mathematical program

$$\text{(W)}\ \underset{t_i \geq 0}{\text{minimize}}\ \sum_{i=1}^{n} w(t_i)$$
$$\text{subject to}$$
$$\prod_{i=1}^{n} r(t_i) \geq r_0.$$

2.2 Reliability maximization problem (R): The second objective is to maximize wing reliability for a specified upper weight limit $w_0 > 0$ by selecting a thickness for each wing panel. The reliability problem (R) is

$$\text{(R)}\ \underset{t_i \geq 0}{\text{maximize}}\ \prod_{i=1}^{n} r(t_i)$$
$$\text{subject to}$$
$$\sum_{i=1}^{n} w(t_i) \leq w_0.$$

2.3 Tradeoff between weight and reliability (P): The third objective is a tradeoff between reliability and weight requirements. Aircraft engineers try to build both reliable and light aircraft. Unfortunately, the goals of maximizing reliability and minimizing weight conflict with each other since increasing reliability increases thickness thereby increasing weight as opposed to decreasing weight, which decreases thickness thereby decreasing reliability. We formulate a multi-objective problem (P) that yields a tradeoff known as a Pareto optimal solution illustrated in Winston (Ref. 8), which is a nondominated feasible point. More precisely, a feasible point $(t_1^*,...,t_n^*)$ to (P) is nondominated if and only if there does not exist another feasible point $(\bar{t}_1,..., \bar{t}_n)$ such that $(\bar{t}_1,..., \bar{t}_n)$ is at least as good as $(t_1^*,...,t_n^*)$ for every objective function of (P) and is strictly better than $(t_1^*,...,t_n^*)$ for at least one objective function. We write this multiple-objective Pareto optimization problem (P) as

$$(\mathrm{P}) \begin{cases} \underset{t_i \ge 0}{\text{minimize}} \; \sum_{i=1}^{n} w(t_i) \\ \underset{t_i \ge 0}{\text{maximize}} \prod_{i=1}^{n} r(t_i) \end{cases}$$

subject to

$$\prod_{i=1}^{n} r(t_i) \ge r_0$$

$$\sum_{i=1}^{n} w(t_i) \le w_0.$$

3. Dynamic programming solution approach

In our problems, each major component upper skin, lower skin and substructure has ten different panels totaling to thirty panels across the wing structure. Figures 4, 5, and 6 illustrate the panels to be analyzed for these three components of aircraft wing.

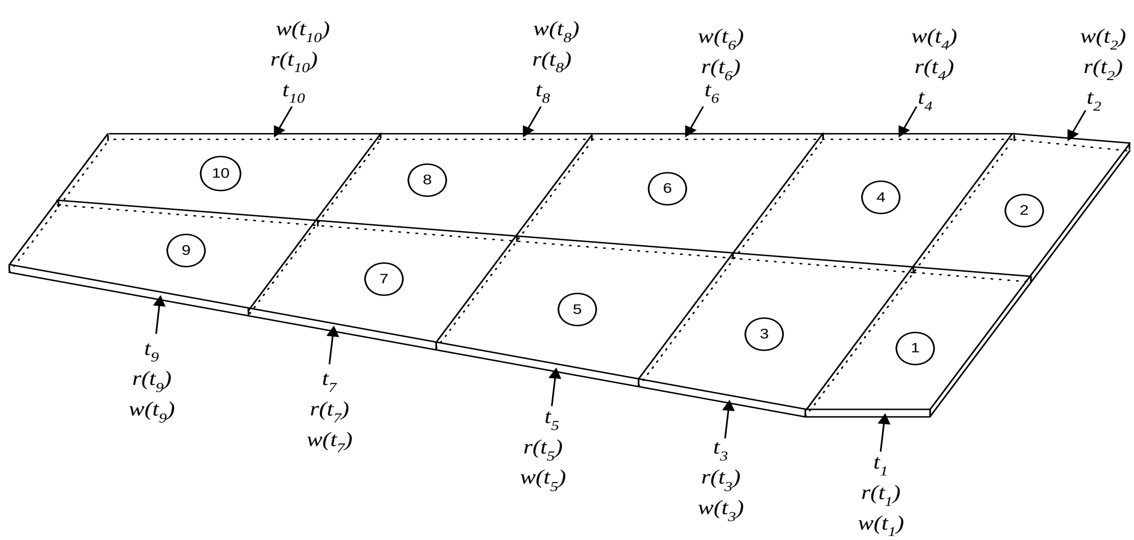

Figure 4. Upper skin

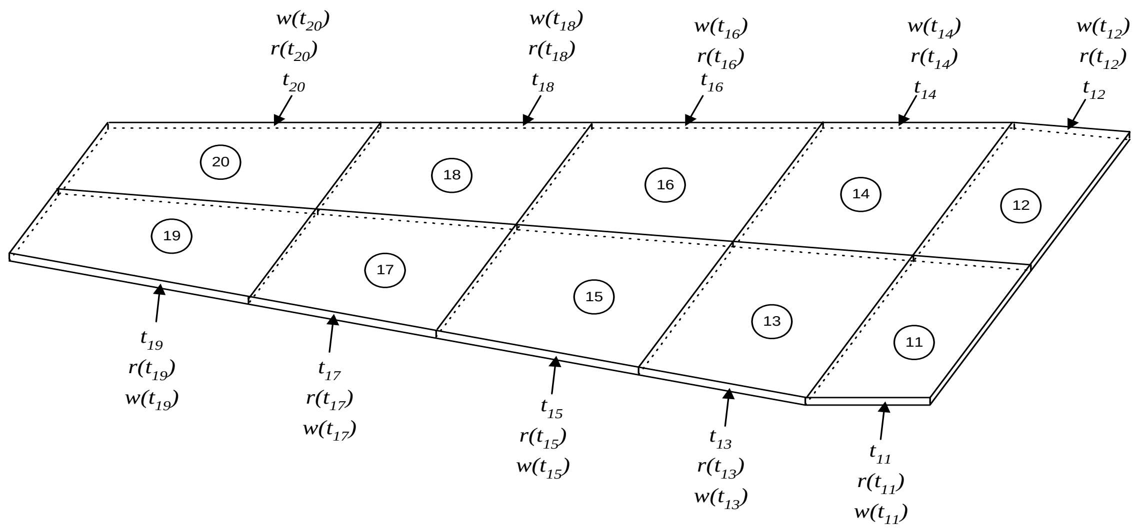

Figure 5. Lower skin.

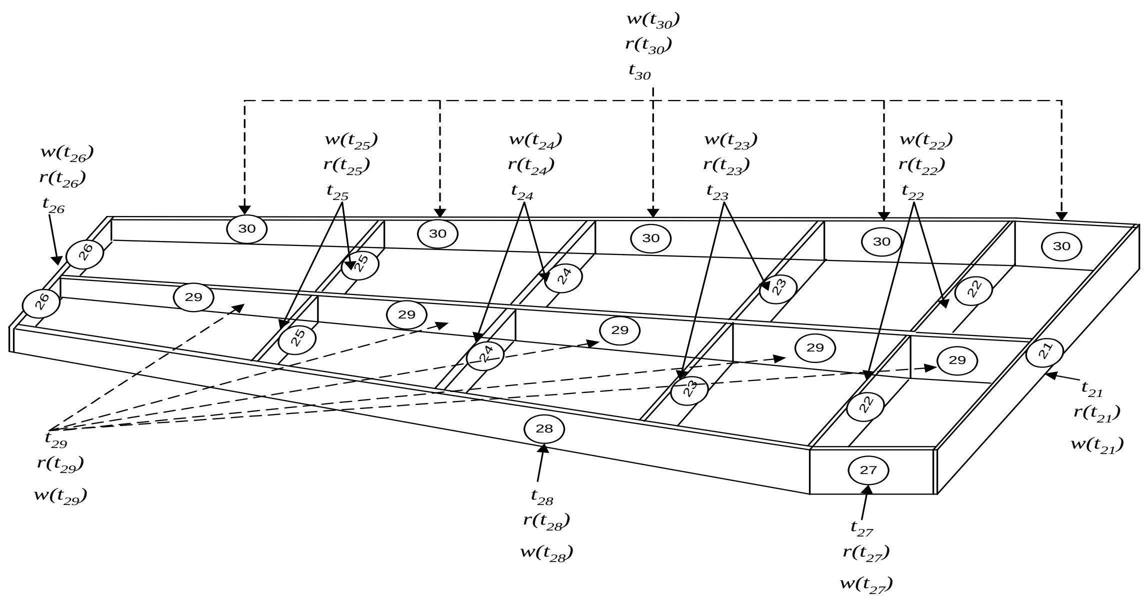

Figure 6. Substructure

For each panel, fourteen different thicknesses were chosen after a thorough investigation regarding their significance to achieving a reliable aircraft wing design. These fourteen different thicknesses are assumed to represent adequately an entire panel for the design purposes. Material strength, operational damage, manufacturing defects, moisture absorption, and gust were incorporated into the NGCAD probabilistic design program to give a predicted structural reliability of the wing box, per thickness, for each panel. Table 1 shows the resulting reliability and weight associated with different thicknesses for panel one with baseline thickness of 0.2 inches.

Table 1. Reliability and weight of panel 1 for different thicknesses

#	Thickness ratio	Reliability	Weight (lbs.)
1	1.20	0.999999995672	5.06
2	1.15	0.999999984471	4.85
3	1.10	0.999999937039	4.64
4	1.05	0.999999759895	4.43
5	1.00	0.999998691860	4.22
6	0.95	0.999992878710	4.01
7	0.90	0.999956177500	3.80
8	0.87	0.999847181000	3.67
9	0.85	0.999757110000	3.59
10	0.84	0.999648429000	3.54
11	0.83	0.999522932000	3.50
12	0.82	0.999318171000	3.46
13	0.81	0.998962640000	3.41
14	0.80	0.998571180000	3.38

Problems (W) and (R) are generalized resource allocation problems with a single constraint. Such problems can be efficiently solved by dynamic programming when the variables are discrete (Ref. 9). Therefore, we use this method to find approximate solutions to these problems by limiting ourselves to a finite number of thickness ratios for each panel as illustrated in Table 1 for panel 1. We then illustrate our approach by solving problem (W). The dynamic programming formulation corresponding (W) is shown in Table 2.

Table 2. Features of dynamic programming formulation corresponding (W)

Number of stages	30
States per stage	Unallocated reliability
Decision variables	Thickness, *t*
Return variables	Weight, *w*

Since our example has thirty stages, with each stage having fourteen possible thicknesses, to make problem (W) computationally tractable we decompose it into the following M subproblems for suitably chosen M and $n_1, \ldots, n_{M-1}, n_M = n$:

Subproblem (W_1) Subproblem (W_2) … Subproblem (W_M)

$$\underset{t_i \geq 0}{\text{minimize}} \sum_{i=1}^{n_1} w(t_i) \qquad \underset{t_i \geq 0}{\text{minimize}} \sum_{i=n_1+1}^{n_2} w(t_i) \qquad \ldots \qquad \underset{t_i \geq 0}{\text{minimize}} \sum_{i=n_{M-1}+1}^{n} w(t_i)$$

$$\text{s.t.} \prod_{i=1}^{n_1} r(t_i) \geq r_1 \qquad \text{s.t.} \prod_{i=n_1+1}^{n_2} r(t_i) \geq r_2 \qquad \ldots \qquad \text{s.t.} \prod_{i=n_{M-1}+1}^{n} r(t_i) \geq r_M \,,$$

where $r_0 = \prod_{j=1}^{M} r_j$.

Similarly we decompose problem (R) into

$$\underline{\text{Subproblem } (R_1)} \qquad \underline{\text{Subproblem } (R_2)} \qquad \dots \qquad \underline{\text{Subproblem } (R_M)}$$

$$\underset{t_i \ge 0}{\text{maximize}} \prod_{i=1}^{n_1} r(t_i) \qquad \underset{t_i \ge 0}{\text{maximize}} \prod_{i=n_1+1}^{n_2} r(t_i) \qquad \dots \qquad \underset{t_i \ge 0}{\text{maximize}} \prod_{i=n_{M-1}+1}^{n} r(t_i)$$

$$\text{s.t.} \sum_{i=1}^{n_1} w(t_i) \le w_1 \qquad \text{s.t.} \sum_{i=n_1+1}^{n_2} w(t_i) \le w_2 \qquad \dots \qquad \text{s.t.} \sum_{i=n_{M-1}+1}^{n} w(t_i) \le w_M ,$$

where $w_0 = \sum_{j=1}^{M} w_j$.

The above decompositions are established in the following two results proved in Chung (Ref. 10).

Result 3.1. Let $t_1^*(r_1)$, $t_2^*(r_1),\dots,t_{n_1}^*(r_1)$, $t_{n_1+1}^*(r_2)$, $t_{n_1+2}^*(r_2),\dots,t_{n_2}^*(r_2),\dots,t_{n_{M-1}+1}^*(r_M)$, $t_{n_{M-1}+2}^*(r_M),\dots,t_n^*(r_M)$ be optimal for $(W_1),\dots,(W_M)$ and r_1, $r_2,\dots,r_M$, respectively. Then the optimal solution to

$$\underset{r_j \ge 0}{\text{minimize}} \left[\sum_{i=1}^{n_1} w\left(t_i^*(r_1)\right) + \sum_{i=n_1+1}^{n_2} w\left(t_i^*(r_2)\right) + \dots + \sum_{i=n_{M-1}+1}^{n} w\left(t_i^*(r_M)\right) \right]$$

$$\text{s.t.} \prod_{j=1}^{M} r_j = r_0$$

solves (W).

Result 3.2. Let $t_1^*(w_1)$, $t_2^*(w_1),\dots,t_{n_1}^*(w_1)$, $t_{n_1+1}^*(w_2)$, $t_{n_1+2}^*(w_2),\dots,t_{n_2}^*(w_2),\dots,t_{n_{M-1}+1}^*(w_M)$, $t_{n_{M-1}+2}^*(w_M)$, $\dots,t_n^*(w_M)$ be optimal for problems (R_1), …, (R_M) and w_1, $w_2,\dots,w_M$, respectively. Then the solution to

$$\underset{w_j \ge 0}{\text{maximize}} \left[\prod_{i=1}^{n_1} r\left(t_i^*(w_1)\right) \times \prod_{i=n_1+1}^{n_2} r\left(t_i^*(w_2)\right) \times \dots \times \prod_{i=n_{M-1}+1}^{n} r\left(t_i^*(w_M)\right) \right]$$

$$\text{s.t.} \sum_{j=1}^{M} w_j = w_0$$

solves (R).

4. Example

Result 1 is now used to solve the minimizing weight problem with thirty locations and fourteen different thicknesses per location. Let r_0 = 0.99999, n = 30, and M = 6. We solve (W) by decomposing it as shown in Table 3. Thus (W) becomes

$$(E_1) \quad \underset{t_i \geq 0}{\text{minimize}} \left[\sum_{i=1}^{5} w(t_i) + \sum_{i=6}^{10} w(t_i) + \sum_{i=11}^{15} w(t_i) + \sum_{i=16}^{20} w(t_i) + \sum_{i=21}^{25} w(t_i) + \sum_{i=26}^{30} w(t_i) \right]$$

$$\text{s.t.} \quad \prod_{i=1}^{5} r(t_i) \times \prod_{i=6}^{10} r(t_i) \times \prod_{i=11}^{15} r(t_i) \times \prod_{i=16}^{20} r(t_i) \times \prod_{i=21}^{25} r(t_i) \times \prod_{i=26}^{30} r(t_i) \geq r_0 .$$

Table 3. Decomposition of example

Subproblems	Locations
(W_1)	locations 1 to 5
(W_2)	locations 6 to 10
(W_3)	locations 11 to 15
(W_4)	locations 16 to 20
(W_5)	locations 21 to 25
(W_6)	locations 26 to 30

The minimum weights for each subproblem will be calculated for various reliability levels. Table 4 and Table 5 show partial data for subproblem (W_1) and (W_2) respectively, where the reliability varies from 0.999 to 0.9999999. After each subproblem is solved, the minimum weights for subproblems (W_1) & (W_2) is used to compute the minimum weights for the combined problems by varying the reliability levels. In other words, the minimum weight for locations 1 - 10 is obtained for various reliabilities. Table 6 shows some of these minimum weights. This logic is also applied to (W_3) & (W_4), as well as (W_5) & (W_6). For subproblems (W_1) & (W_2), (W_3) & (W_4), and (W_5) & (W_6), 290, 255, and 80 minimum weights are calculated, respectively.

Finally, the complete problem (E_1) for locations 1 – 30 becomes

$$(E_2) \quad \underset{r_j \geq 0}{\text{minimize}} \; [\sum_{j=1}^{3} w^*\left(r_j\right)]$$

$$\text{s.t.} \quad \prod_{j=1}^{3} r_j \geq r_0, \text{ where } r_0 = 0.99999.$$

We have only three stages associated with subproblems (W_1) & (W_2), (W_3) & (W_4), and (W_5) & (W_6). Dynamic programming will be applied to these three subproblems. To illustrate, we give the data for (W_1) & (W_2).

Table 4. Choices of subproblem (W_1)

Choices	Reliability level	Optimal weight (lbs.)
1	0.9999054	30.70230
2	0.9999109	30.80540
3	0.9999130	30.82888
...	...	...
83	0.99999996	39.21233
84	0.99999997	39.62026

Table 5. Choices of subproblem (W_2)

Choices	Reliability level	Optimal weight (lbs.)
1	0.99994428	51.7205
2	0.9999456	51.7567
3	0.9999462	51.8049
...	...	...
211	0.99999999993	71.1159
212	1.00000000000	74.3483

Table 6. Results of subproblems (W_1) and (W_2)

Choices	Reliability level r_1	Weight $w^*(r_1)$ (lbs.)
1	0.999999980	114.59
2	0.999999979	108.33
3	0.999999976	108.12
...	...	...
289	0.999851000	82.46
290	0.999849000	82.42

The resulting total minimal weight is 249.93 lbs. Table 7 shows the choices for each panel. Three different designs yield a minimum wing weight without violating the reliability constraint. The maximum reliability among these three selections is 0.9999905. An engineer would undoubtedly select this design. The maximum reliability problem (R) can be similarly solved utilizing Result 3.2.

Table 7. Choices for each panel

Panel	Choice 1 Thickness (inches)	Choice 2 Thickness (inches)	Choice 3 Thickness (inches)
1	1.10	1.05	1.05
2	1.05	1.05	1.10
3	0.90	0.95	0.90
4	0.90	0.90	0.90
5	0.95	0.95	0.95
6	0.87	0.87	0.87
7	0.87	0.87	0.87
8	0.87	0.87	0.87
9	0.85	0.85	0.85
10	0.87	0.87	0.87
11	0.80	0.80	0.80
12	0.80	0.80	0.80
13	0.80	0.80	0.80
14	0.80	0.80	0.80
15	0.80	0.80	0.80
16	0.80	0.80	0.80
17	0.80	0.80	0.80
18	0.80	0.80	0.80

19	0.80	0.80	0.80
20	0.80	0.80	0.80
21	0.80	0.80	0.80
22	0.80	0.80	0.80
23	0.80	0.80	0.80
24	0.80	0.80	0.80
25	0.80	0.80	0.80
26	0.80	0.80	0.80
27	0.80	0.80	0.80
28	0.80	0.80	0.80
29	0.80	0.80	0.80
30	0.80	0.80	0.80

5. Tradeoffs

To solve problem (P), we solve (W) for various r_0 as in the above section to give a minimum weight $w_0(r_0)$. Then each such pair $(w_0(r_0), r_0)$ is a Pareto optimum as discussed in Winston (Ref. 8). The parameterized set of problems $\{P(r_0) : 0 < r_0 < 1\}$ is thus

$$\{P(r_0)\} \quad \underset{r_j \geq 0}{\text{minimize}} \; [\sum_{j=1}^{3} w^*\left(r_j\right)]$$

$$\text{s.t.} \; \prod_{j=1}^{3} r_j \geq r_0 \,, 0 < r_0 < 1 \,.$$

Solving $\{P(r_0)\}$ for values of r_0 in [0.9999, 0.99999999] yields the Pareto optimal solutions shown in Figure 7, which is called the tradeoff curve in that reliability region. Any point on the curve is nondominated for (P). Among these possible solutions, one could be selected according to further relevant criteria besides minimum weight and maximum reliability.

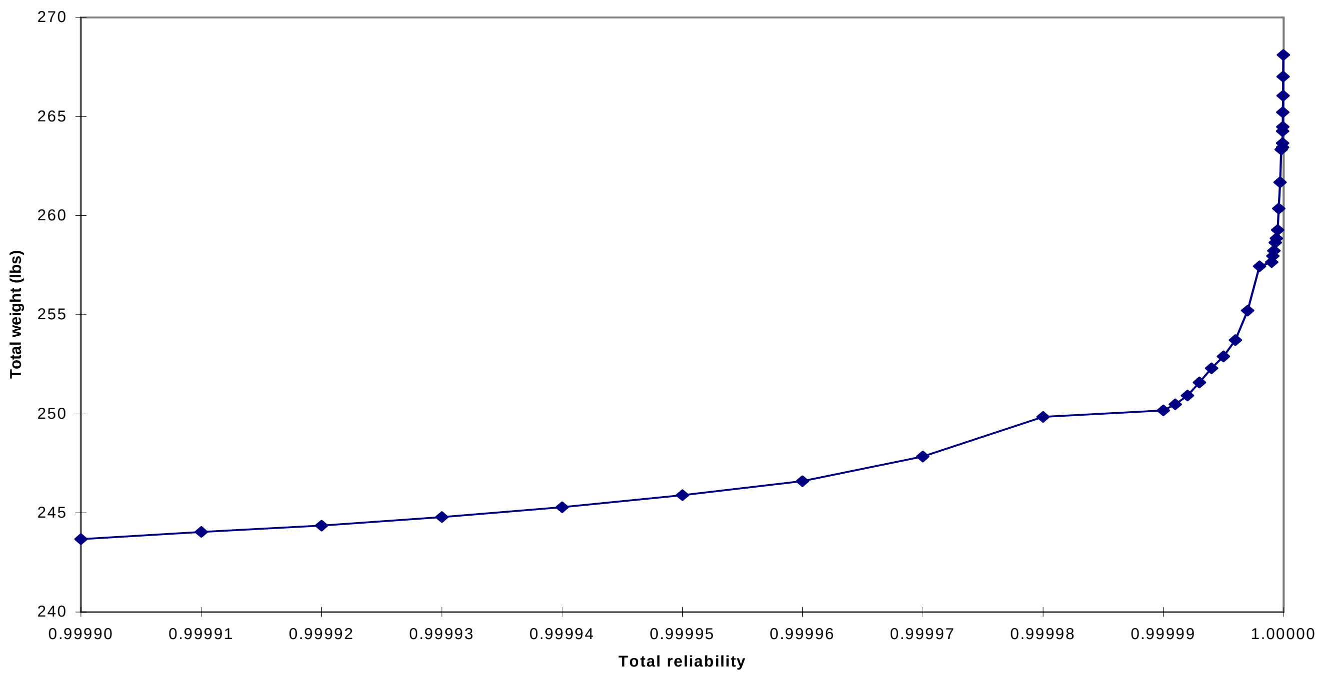

Figure 7. Tradeoff curve

6. Conclusions

A discrete dynamic programming approach was presented for obtaining the optimal thickness of aircraft wings. The criteria were (1) minimizing a total wing's weight while satisfying the FAA safety regulations (reliability requirements), (2) maximizing a wing's reliability within weight limitations, and (3) determining tradeoff designs between these two criteria. An example was presented to illustrate our method. There are two principle directions for further research. First, a precise analytical approach for determining the number and position of the wing locations could be developed. Second, despite Results 3.1 and 3.2, the memory requirements and number of calculations are extremely large. Therefore, one should consider implementing the dynamic programming approach in a parallel processing environment.

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