A Stochastic Programming Approach for Integrated Nurse Staffing and Assignment

Prattana Punnakitikashem[1,*], Jay M. Rosenberger[1], Deborah Buckley Behan[2]

[1]Department of Industrial and Manufacturing Systems Engineering
The University of Texas at Arlington
Arlington, TX 76019-0407, USA
prattana.punnakitikashem@uta.edu
jrosenbe@uta.edu

[2]School of Nursing
The University of Texas at Arlington
Arlington, TX 76019-0017, USA
dbehan@uta.edu

Abstract

The renowned nursing shortage has attracted much attention from national level law makers, state legislatures, commercial organizations, and researchers due to its direct impact on the quality of patient care. High workloads and undesirable schedules are two major issues that cause nurses' job dissatisfaction. The focus of this paper is to find nondominated solutions to an integrated nurse staffing and assignment problem that minimize two criteria, which are excess workload on nurses and nurse staffing cost. Initially, we present a stochastic integer programming model with an objective to minimize excess workload subject to a hard budget constraint. Accordingly, we develop three solution approaches, which are Benders' decomposition, Lagrangian relaxation with Benders' decomposition, and nested Benders' decomposition. We vary the maximum allowable staffing cost in the budget constraint in Benders' decomposition and nested Benders' decomposition, and we relax the budget constraint and penalize staffing cost in the Lagrangian relaxation with Benders' decomposition approach. We collect nondominated bicriteria solutions from the algorithms. We demonstrate the effectiveness of the model and algorithms with a computational study based upon data from two medical-surgical units at a Northeast Texas hospital. A float assignment policy is also evaluated. Finally, areas of future research are discussed.

Keywords: nurse staffing, nurse rescheduling, nurse rerostering, nurse scheduling, nurse assignment, stochastic programming, bicriteria, Benders' decomposition, Lagrangian relaxation, nested Benders' decomposition.

* This research was part of Prattana Punnakitikashems Ph.D. Dissertation [62]

1 Introduction

The renowned nursing shortage has attracted much attention from national level law makers, state legislatures, commercial organizations, and researchers due to its direct impact on the quality of patient care [4, 12, 26, 36, 71]. The decline in enrollments in registered nurse degree programs, the increasing average age of working registered nurses, nurse burnout, and job discontentment have also intensified the nursing shortage problem [4]. High workloads and undesirable schedules are two major issues that cause nurses' job dissatisfaction [4, 27, 70]. Powers [61] observed that excessive workload decreases the quality of patient care. Many states have taken actions to cope with the shortage to ensure patient safety. For example, California has regulated mandatory nurse-to-patient ratios [23, 78]. To satisfy patient care demands, hospital administrations are obligated to employ other expensive staffing resources, such as part-time nurses, agency nurses, overtime nurses, etc. Since nurse staffing costs account for over 50% of a hospital expenditures [42], healthcare costs are continuously increasing driven by an ongoing severe shortage of nurses. Consequently, nurse staffing has become one of the most attractive research areas.

A nurse scheduling problem involves creating a schedule for nurses. The schedule determines nursing staffs that work for a shift on a given day subject to patient care demands, budget, nurses' requirements, hospital policies, and legal regulations. We refer to *nurse scheduling* or *nurse rostering* as a mid-term scheduling, which occurs 4-6 weeks before a shift. A nurse supervisor forecasts patient demands over a time horizon. The supervisor uses a census matrix to determine the number and level of nurses needed. Nurses are allowed to request their preferred schedules. Then, the supervisor creates a schedule and posts it two weeks in advance to a shift. Cheang et al. [24] distinguished three major models of nurse scheduling, which are scheduling a nurse to work on a certain shift on a certain day, on a certain task, and on a certain shift pattern. Most nurse rostering literature can be found in Burke et al. [20], Cheang et al. [24], and Sitompul and Randhawa [69]. They summarized the nurse rostering model and the solution methodologies from the 1960's until 2004. Nurse rostering models and solution approaches included linear and integer programming [1, 32, 37, 40, 46, 47, 48, 65, 72, 74, 75, 76], goal programming/multi-criteria approaches [6, 7, 13, 21, 25, 34, 39, 51, 59, 60], artificial intelligence methods [25, 45, 55, 56, 67], heuristics [5, 17, 43], and metaheuristic, i.e., simulated annealing [18, 38], tabu search [19, 29, 30], genetic algorithms [2, 3, 22, 41].

We refer to *nurse staffing*, *nurse rescheduling*, or *nurse rerostering* as a short-term nurse scheduling that occurs 90-120 minutes prior to an upcoming shift. Given a mid-term nurse schedule, the nurse supervisor reviews the nurse schedules based upon activities of the previous shift, activities in other units, and either a census matrix or a patient classification system. When a unit is understaffed, the supervisor recruits additional nurses to cover patient care demands. On the other hand, if the supervisor has surplus nurses, then they either float to other units or take the day off without pay. Based upon nurse staffing, a charge nurse assigns each patient to a nurse at the beginning of a shift. Workload balance is one important consideration for nurse assignment. We refer to a nurse's *workload* as the amount of time required to care for her patients over a time period, and *excess workload* is the difference between the workload and the time available for care.

Nurse staffing has a direct impact on nurse-to-patient assignment, nurse workload, and the quality of care for patients [4, 44]. Thus far, nurse staffing models have only considered nurse scheduling and ignored nurse-to-patient assignment. Taking patient information into consideration, models will be able to meet patients' needs while utilizing nursing staffs efficiently. Furthermore, most of the existing models proposed in the optimization literature are deterministic, which exclude uncertainty in patient care. Patient care is stochastic in nature due to its fluctuations during the shift and its enormous variation. In this paper, we accommodate these issues that have been neglected in academic literature within our model. We develop a new integrated nurse staffing and assignment model with a stochastic integer programming method to capture randomness in patient care.

The structure of the remaining paper is organized as follows. In Section 2, we describe nurse staffing and assignment with relevant literature. In Section 3, we present a stochastic programming formulation for the new integrated nurse staffing and assignment model. The underlying assumptions, decision variables, and parameters, which are used in the model are also described in Section 3. In Section 4, we present three alternative solution approaches, which are Benders' decomposition, Lagrangian relaxation with Benders' decomposition, and nested Benders' decomposition. As we focus on finding solutions that minimize both excess workload and staffing cost objectives, we vary the maximum allowable staffing cost in the hard budget constraint in the Benders' decomposition and the nested Benders' decomposition approaches. In the Lagrangian relaxation with Benders' decomposition approach, we relax the budget constraint and penalize the staffing cost in the objective function. We use our algorithm approaches to find several bi-criteria nondominated nurse staffing and assignment solutions. In Section 5, computational results are provided based upon real data from a Northeast Texas hospital. Many solutions with different tradeoffs between staffing cost and excess workload for nurses (or implicit quality of patient care) are presented to help the nurse supervisor determine the right staffing policy for an upcoming shift. Moreover, we evaluate a floating nurses policy. Finally, we discuss conclusions and directions for future research in Section 6.

2 Problem Description and Literature Review

Nurse staffing is an important issue for every hospital for several reasons. In addition to its direct effect on patient care and hospital budgeting, nurse staffing is a routine performed daily by almost all units throughout a year. In most hospitals, nurse staffing is performed on three shifts per weekday, and these shifts include a day shift (7:00-15:00), an evening shift (15:00-23:00), and a night shift (23:00-7:00). Two shifts occur per weekend day, which are a day shift (6:00-18:00), and a night shift (18:00-6:00). Nurse staffing typically occurs 90-120 minutes prior to a shift. A nurse supervisor reevaluates a mid-term staffing based upon daily disruption from nurses, patients, and hospital administrations. The nurse supervisor considers the activities of the previous shift, activities in other units, activities in emergency rooms, activities in doctors' offices, and either a census matrix or a patient classification system. Uncertainty from nurses such as absenteeism, sick leave, or taking a personal day off is also taken into consideration. A few hours before a shift, a nurse supervisor has most patient information including patient diagnoses, patient acuities, census data, patient rooms, admission dates, estimated discharge dates, and special requirements for specialty nurses.

We consider scheduled nurses, float nurses, PRN nurses, overtime nurses, and agency nurses in nurse staffing. We refer to *scheduled nurses* or *regular nurses* as nurses who are scheduled to work for particular units in a given shift on a given day. *PRN nurses* are called as needed to work when there is a shortage in a shift. They primarily work on one unit. After the schedule is made, they can schedule themselves to work if there are holes needed to be filled. *On-call nurses* cannot go further than about a 30-mile radius from a hospital. They must carry a pager, and be ready to go to the hospital when they are called. If they are called to work, they will get paid the full amount for their services. Given that on-call nurses usually work in gastrointestinal laboratories, operating rooms, or other special areas, and they are not assigned to patients, we exclude on-call nurses from our model. *Agency nurses* are available to call in from agency nurse services. *Overtime nurses* are nurses who work in the current shift and will work consecutively in the next shift. *Float nurses* are nurses who are scheduled to work in their home units or to a float pool. When help is needed, float nurses are reassigned to the unit that is short for a shift. Typically, they float from overstaffed units to understaffed units to cover patient care demands. Float nurses are increasingly used to ensure patient safety in hospital units. However, float nurses suffer from unfamiliarity with unit environments, unit routines, staffs, and patient complications [66].

Based upon the mid-term schedule, the nurse supervisor reviews the scheduled nurses and up-to-date nurse information. If there are more nurses than needed, she lets voluntary surplus scheduled nurses choose one of two following options: float to other units or take that day off without pay. As a nurse takes a day off, one of the vacation, personal, holiday, or unpaid leave days is used. Typically, a nurse prefers her shift canceled over being floated. When there is a shortage of nurses for the upcoming shift, the nurse supervisor recruits additional nurses from the following priorities:

1. voluntary excess nurses from other units,
2. float nurses,
3. PRN nurses,
4. mandatory overtime nurse,
5. agency nurses.

Given that excess nurses volunteer to work in other units, it is reasonable to consider them as float nurses since this information is known in advance. Patient classification systems are the most sophisticated technology for nurse staffing. These systems group patients into one of several categories. They estimate how many times certain tasks will be performed in caring for a patient in each category. Using these estimates and the expected time required to perform each task, the systems determine the amount of time to care for a typical patient. As patients are admitted into the unit, the system classifies these patients, and the nurse supervisor use the estimated patient care to determine how many nurses are needed for the shift. As a patient's condition changes, he may be given a new patient classification.

According to the optimization literature, most research on nurse staffing has emerged recently. Abernathy et al. [1] integrated nurse scheduling and nurse staffing. Warner et al. [74] addressed a need for short-term staffing when unexpected absences occur. Siferd and Benton [68] developed a stochastic model based upon the patients in a unit to determine how many nurses are required for the shift. Nevertheless, the different sets of skills among the nurses are ignored. Bard and Purnomo [8, 9, 10] presented deterministic integer programming models for daily nurse rescheduling. Moz and Pato described nurse rerostering as a multi-commodity flow problem with an objective to minimize the difference between the original and new schedules [48]. They also constructed a genetic algorithm to solve this problem [49]. Walts and Kapadia [73] presented a patient classification system and optimization model while Yankovic and Green [79] used a queueing model to determine the level of staffing to meet the required workload level. Vericourt and Jennings [28] employed a queuing model to investigate nurse-to-patient ratios mandated by California, and they proposed two heuristic staffing policies. They also described that hospitals with no nurse-to-patient ratio policy can provide consistently good quality of care for every unit. Wright et al. [78] developed a bicriteria nonlinear integer programming model to evaluate the impact of nurse-to-patient ratios on schedule cost and nurse desirability.

Recently, float nurses have been widely used due to the shortage of nurses. It involves allocation of nurses from one unit or from a float pool of nurses to another unit to meet patient care demand. Trivedi and Warner [72] developed a branch and bound algorithm for allocating nurses in a float pool. Wright et al. [77] presented nurse rescheduling and their model also staffed cross-trained float pool nurses to an understaffed unit. The literature included only allocating nurses from a float pool to one of several units in a hospital, none of which considered floating nurses from one unit to other units.

Given a set of nurses from nurse staffing, a charge nurse assigns each patient to a nurse at the beginning of a shift, which is referred as *nurse assignment*. In general, nurse assignment is performed approximately 30 minutes prior to a shift. Modern patient classification systems partition the set of patients into groups, and each group is assigned to a nurse [58]. Mullinax and Lawley [50] developed a deterministic integer programming model that assigns patients to nurses in a neonatal intensive care unit. Rosenberger et al. [64] presented a deterministic model for patient assignment. Punnakitikashem et al. [63] developed a stochastic programming to assign nurses to patients with an objective to minimize excess workload on nurses. Nevertheless, no one has ever integrated the nurse staffing with the nurse assignment.

2.1 Contributions

This paper focuses on integrating nurse staffing and assignment. Since short-term nurse staffing is considered, it is unnecessary to account for nurse preferences, which are normally included in mid-term scheduling. We present an extension of the stochastic programming model for patient assignment (SPA) from Punnaktikashem et al. [63] by incorporating the nurse staffing decision into the assignment model. Our model determines a staff and patient assignments for nurses that minimize their excess workload while satisfying a budget and considering uncertain patient care. By varying the maximum allowable staffing cost, we can search for nondominated solutions that minimize both excess workload and cost. The model can be viewed as either a general resource allocation model or a personnel scheduling model, which can be employed in other applications.

The contribution of this paper includes the first model to integrate short-term nurse staffing and assignment. The model provide good solutions for several units simultaneously. The model is also a stochastic program addressing several important issues that are ignored in academic literature and patient classification systems.

- Patient Uncertainty: Traditional nurse assignment models ignore uncertainty. Because of the enormous variance in patient care, the stochastic programming model that considers uncertainty provides more robust solutions.
- Fluctuations in Patient Care: Traditional models ignore fluctuations in patient care during the shift. Some patients, such as expectant mothers, require minimal care for part of a shift but require significant care at other times during the shift. The stochastic programming model considers when patients require care.
- Differences in Nurses: Traditional models ignore the different skills of the nurses. Many of them use a targeted amount of time to perform certain tasks instead of an average time to complete the task. Some targets may be

realistic for some nurses but unrealistic for others. The stochastic programming model considers the skills of each nurse individually.

- Patient Information: Traditional nurse staffing models ignore patients information. Most of them consider only a number of nurses needed or patient care demand by hours. The stochastic programming model considers each individual patient, and determines a patient's required care from a patient's diagnosis and location.

In addition to the formulation of a new integration of nurse staffing and assignment, this paper develops three algorithmic approaches, which are (1) Benders' decomposition, (2) Lagrangian relaxation with Benders' decomposition, and (3) nested Benders' decomposition, to solve it. Unlike the traditional nurse staffing models with single objective function, this paper focuses on finding solutions that minimize two criterias, which are excess workload on nurses and staffing cost. Accordingly, we alter the right-hand side to the budget constraint in Benders' decomposition and nested Benders' decomposition. We relax the budget constraint and penalize staffing cost in the objective function resulting in the new Lagrangian relaxation and Benders' decomposition approach. Moreover, instead of providing only a single solution like the traditional models, this paper focuses on collecting nondominated solutions in the algorithms to provide a nurse supervisor several quality nurse staffing and assignment solutions. We demonstrate the efficiency of these approaches with a computational study based upon real data from two medical-surgical units at the Northeast Texas hospital. Lastly, the Lagrangian relaxation with Benders' decomposition algorithm is used to evaluate a float nurse assignment policy. Our model allows allocation of nurses from one unit to other units. We demonstrate how a nurse administrator can use our model to assess float assignment policies between floating nurses from one unit to another unit versus not floating them. In summary, this paper fulfills nurse staffing future directions pointed out by Burke et al. [20] in the following issues: constructing a multi-criteria model, decomposing large problems into smaller problems, and developing and evaluating a model based upon real world data.

3 Stochastic Programming Model

In this paper, we made the following reasonable assumptions:

Assumption 1: The number of nurses in each type who are available to provide services to each hospital unit is known.

Assumption 2: The qualifications and specialties of nurses in each type are known. A list of qualified nurses who can provide care to each patient is known prior to staffing.

Assumption 3: The cost function is linear.

In the model, let J be the set of units and P be the set of patients. Based on a mid-term schedule, let N denote the set scheduled nurses, including full-time nurses and float nurses assigned to work for a shift. Let R, O, and A be the sets of PRN nurses, overtime nurses, and agency nurses for a shift, respectively. Let **N** be the set all nurses including full-time nurses, float nurses, PRN nurses, overtime nurses, and agency nurses for a shift; that is, $\mathbf{N} = N \cup R \cup O \cup A$. For each unit $j \in J$, let $P(j)$ be the set of patients who is in unit j. For each unit $j \in J$, let $N(j)$ be the set of full-time nurses and float nurses who are scheduled to work for a shift in unit j. Let $R(j)$, $O(j)$, and $A(j)$ be the sets of PRN nurses, overtime nurses, and agency nurses qualified to work in unit j, respectively. For each patient $p \in P$, let $N(p)$, $R(p)$, $O(p)$, and $A(p)$ be the sets of scheduled nurses, PRN nurses, overtime nurses, and agency nurses who can be assigned to patient p, respectively. For each patient $p \in P$, let $\mathbf{N}(p)$ be the set of nurses which can be assigned to patient p. For each nurse $n \in \mathbf{N}$, let $P(n)$ be the set of patients that can be assigned to nurse n; that is, $P(n) = \{p \in P | n \in \mathbf{N}(p)\}$.

For each unit $j \in J$, and nurse $n \in N(j)$, let *scheduled nurse variable*

$$Y_{nj} = \begin{cases} 1 & \text{if a scheduled nurse } n \in N(j) \text{ is assigned to work for a shift for unit } j \in J, \\ 0 & \text{otherwise.} \end{cases}$$

For each nurse $n \in N$, let *cancellation variable*

$$Y_n^c = \begin{cases} 1 & \text{if a scheduled nurse } n \in N \text{ is canceled for her shift,} \\ 0 & \text{otherwise.} \end{cases}$$

For each unit $j \in J$, and nurse $n \in R(j)$, let *PRN staffing variable*

$$Y_{nj}^r = \begin{cases} 1 & \text{if a PRN nurse } n \in R(j) \text{ is assigned to work for a shift in unit } j \in J, \\ 0 & \text{otherwise.} \end{cases}$$

For each unit $j \in J$, and nurse $n \in O(j)$, let *overtime staffing variable*

$$Y_{nj}^o = \begin{cases} 1 & \text{if an overtime nurse } n \in O(j) \text{ is assigned to work for a shift in unit } j \in J, \\ 0 & \text{otherwise.} \end{cases}$$

For each unit $j \in J$, and nurse $n \in A(j)$, let *agency staffing variable*

$$Y_{nj}^a = \begin{cases} 1 & \text{if an agency nurse } n \in A(j) \text{ is assigned to work for a shift in unit } j \in J, \\ 0 & \text{otherwise.} \end{cases}$$

There are costs associated with hiring nurses. For each unit $j \in J$ and each nurse $n \in N(j)$, let c_{nj}^s be the cost of staffing full-time nurse or float nurse n to work in unit j. For each nurse $n \in N$, let c_n^c be the cost of canceling scheduled nurse n. For each unit $j \in J$ and each nurse $n \in R(j)$, $n \in O(j)$, and $n \in A(j)$, let c_{nj}^r, c_{nj}^o, and c_{nj}^a be the cost of hiring PRN nurses, overtime nurses, and agency nurses for unit j, respectively. Let B be a budget for hiring nurses for all units on the shift. All costs and the budget are given as parameters to the model.

For each patient $p \in P$, and nurse $n \in \mathbf{N}(p)$, let *assignment variable*

$$X_{pn} = \begin{cases} 1 & \text{if patient } p \in P \text{ is assigned to nurse } n \in \mathbf{N}(p), \\ 0 & \text{otherwise.} \end{cases}$$

A shift is divided into a set of time periods T. Let Ξ be a set of random scenarios, and for each $\xi \in \Xi$, let ϕ^ξ be the probability that scenario ξ occurs. For each time period $\tau \in T$ and each nurse $n \in \mathbf{N}$, let $A_{\tau ni}^\xi$ be the *total workload variable* representing the amount of workload assigned to nurse n between time durations $m_{\tau ni}$ and $m_{\tau n(i+1)}$ in scenario $\xi \in \Xi$.

We model the penalty for assigning workload to nurses as a monotonically nondecreasing piecewise linear convex function with k pieces. Let $\alpha_{\tau ni}$ denote the marginal penalty of $A_{\tau ni}$ for $1 \leq i \leq k$. Let $m_{\tau n(k+1)}$ be ∞. The penalty is monotonically nondecreasing meaning that $0 = m_{\tau n1} < \ldots < m_{\tau nk}$ and $0 \leq \alpha_{\tau n1} < \ldots < \alpha_{\tau nk}$. Since the penalty function is nondecreasing and piecewise linear convex, an overworked nurse receives more penalty than a nurse with less workload. Thus, the penalty function balances the workload for all nurses. One special case of the penalty function that we used in this paper has $k = 2$, $\alpha_{\tau n1} = 0$, $\alpha_{\tau n2} = 1$, and $m_{\tau n2}$ equal to the duration of the time period τ for each $\tau \in T$ and each $n \in \mathbf{N}$. We refer to the value of variable $A^{\xi}_{\tau n2}$ as the *excess workload* on nurse n in time period τ and scenario ξ. The objective of our model in the computational results in Section 5 is to minimize the expected excess workload on nurses.

Given that the patient care is stochastic in nature, the patient care may vary dramatically throughout the shift. For each patient $p \in P$, each scenario $\xi \in \Xi$, and each $t \in T$, let d^{ξ}_{tp} denote the amount of direct care required by patient p in time period t. For each patient $p \in P$, each scenario $\xi \in \Xi$, and each time period $t \in T$, let g^{ξ}_{tp} denote the amount of indirect care required by patient p at the beginning of time period t until the end of the shift. For each pair of time periods $(t, \tau) \in T \times T$, where $t \leq \tau$, and each nurse $n \in N$, let *indirect workload variable* $G^{\xi}_{t\tau n}$ be the total indirect care that can be performed during or after time period t and is performed in time period τ by nurse n. The amount of direct and indirect care the patients require in each time period under each scenario are given as parameters to the model.

The *extensive form* of the Stochastic Integrated Nurse Staffing and Assignment Model (SINSA) can be formulated as:

$$\min \sum_{\xi \in \Xi} \sum_{n \in \mathbf{N}} \sum_{\tau \in T} \sum_{i=1}^{k} \phi^{\xi} \alpha_{\tau n i} A^{\xi}_{\tau n i}, \tag{1}$$

subject to

$$\sum_{j \in J} \sum_{n \in N(j)} c^s_{nj} Y_{nj} + \sum_{n \in N} c^c_n Y^c_n +$$

$$\sum_{j \in J} \sum_{n \in R(j)} c^r_{nj} Y^r_{nj} + \sum_{j \in J} \sum_{n \in O(j)} c^o_{nj} Y^o_{nj} +$$

$$\sum_{j \in J} \sum_{n \in A(j)} c^a_{nj} Y^a_{nj} \leq B, \tag{2}$$

$$\sum_{j \in J(n)} Y_{nj} + Y^c_n = 1 \qquad \forall n \in N, \tag{3}$$

$$Y_{nj} \geq X_{pn} \qquad \forall n \in N(p), p \in P(j), j \in J, \tag{4}$$

$$Y^r_{nj} \geq X_{pn} \qquad \forall n \in R(p), p \in P(j), j \in J, \tag{5}$$

$$Y^o_{nj} \geq X_{pn} \qquad \forall n \in O(p), p \in P(j), j \in J, \tag{6}$$

$$Y^a_{nj} \geq X_{pn} \qquad \forall n \in A(p), p \in P(j), j \in J, \tag{7}$$

$$Y_{nj} \in \{0,1\} \qquad \forall n \in N(j), j \in J, \tag{8}$$

$$Y^c_n \in \{0,1\} \qquad \forall n \in N, \tag{9}$$

$$Y^r_{nj} \in \{0,1\} \qquad \forall n \in R(j), j \in J, \tag{10}$$

$$Y^o_{nj} \in \{0,1\} \qquad \forall n \in O(j), j \in J, \tag{11}$$

$$Y^a_{nj} \in \{0,1\} \qquad \forall n \in A(j), j \in J, \tag{12}$$

$$\sum_{n \in \mathbf{N}(p)} X_{pn} = 1 \qquad \forall p \in P, \tag{13}$$

$$\sum_{p \in P(n)} g^{\xi}_{tpn} X_{pn} = \sum_{\tau=t}^{|T|} G^{\xi}_{t\tau n} \qquad \forall t \in T, n \in \mathbf{N}, \xi \in \Xi, \tag{14}$$

$$\sum_{p \in P(n)} d^{\xi}_{\tau pn} X_{pn} + \sum_{t=1}^{\tau} G^{\xi}_{t\tau n} = \sum_{i=1}^{k} A^{\xi}_{\tau n i} \qquad \forall \tau \in T, n \in \mathbf{N}, \xi \in \Xi, \tag{15}$$

$$X_{pn} \in \{0,1\} \qquad \forall p \in P(n), n \in \mathbf{N}, \tag{16}$$

$$G^{\xi}_{t\tau n} \geq 0 \qquad \forall t, \tau \in T, t \leq \tau, n \in \mathbf{N}, \xi \in \Xi, \tag{17}$$

$$m_{\tau n(i+1)} - m_{\tau n i} \geq A^{\xi}_{\tau n i} \geq 0 \qquad \forall \tau \in T, 1 \leq i \leq k, n \in \mathbf{N}, \xi \in \Xi. \tag{18}$$

Objective function (1) is to minimize expected excess workload on nurses. Constraint (2) is the *budget constraint*, which ensures that the cost of hiring and canceling nurses does not exceed the budget. For each scheduled nurse $n \in N$, the *cancelation constraints* in set (3) indicate that either she is assigned to work or her shift is canceled. Constraints (4)-(7) are linking constraints between staffing and assignment decision variables. If a nurse is assigned to a patient, then she must be scheduled to work for a shift. Constraints (8)-(12) require the staffing variables be binary. The *nurse assignment constraints* in set (13) ensure that every patient is assigned to a nurse. For each nurse

$n \in \mathbf{N}$, the *indirect care constraints* in set (14) determine the total indirect care performed from the beginning of time period t until the end of the shift. For each time period $\tau \in T$, the *total workload constraints* in set (15) define the total workload of nurse $n \in \mathbf{N}$ containing both direct care and indirect care. Constraint set (16) includes the *binary constraints* for the assignment variables. The nonnegativity constraints in set (17) require the indirect care variables be nonnegative. The upper and lower bounds on the marginal workload variables are provided by constraints (18). For each $\tau \in T, n \in \mathbf{N}, \xi \in \Xi$, the total workload variable $A^{\xi}_{\tau nk}$ has no upper bound because $m_{\tau n(k+1)} = \infty$. For constraints (13)-(18) related to nurse assignment, the unit index j can be neglected because the unit is embedded in the patient information.

The *deterministic equivalent model* for integrated nurse staffing and assignment can be written as follows:

$$\min Q(X), \tag{19}$$

subject to

$$\text{(2)-(13) and (16).} \tag{20}$$

where $Q(X)$ is the *expected second-stage recourse function* defined as:

$$Q(X) = E_{\xi} Q(X, \xi), \tag{21}$$

and

$$Q(X, \xi) = \min \sum_{n \in \mathbf{N}} \sum_{\tau \in T} \sum_{i=1}^{k} \alpha_{\tau ni} A^{\xi}_{\tau ni} \tag{22}$$

subject to

$$\sum_{p \in P(n)} g^{\xi}_{tpn} X_{pn} = \sum_{\tau=t}^{|T|} G^{\xi}_{t\tau n} \qquad \forall t \in T, n \in \mathbf{N}, \tag{23}$$

$$\sum_{p \in P(n)} d^{\xi}_{\tau pn} X_{pn} + \sum_{t=1}^{\tau} G^{\xi}_{t\tau n} = \sum_{i=1}^{k} A^{\xi}_{\tau ni} \qquad \forall \tau \in T, n \in \mathbf{N}, \tag{24}$$

$$G^{\xi}_{t\tau n} \geq 0 \qquad \forall t, \tau \in T, t \leq \tau, n \in \mathbf{N}, \tag{25}$$

$$m_{\tau n(i+1)} - m_{\tau ni} \geq A^{\xi}_{\tau ni} \geq 0 \qquad \forall \tau \in T, 1 \leq i \leq k, n \in \mathbf{N}. \tag{26}$$

4 Algorithmic Approaches

In this section, we present decomposition approaches for solving SINSA, which are Benders' decomposition, Lagrangian relaxation with Benders' decomposition, and nested Benders' decomposition. Among the three decomposition approaches, only Benders' decomposition is an exact approach while the other two approaches, which are Lagrangian relaxation with Benders' decomposition and nested Benders' decomposition, are linear programming relaxations of the original problem. SINSA is viewed as a two-stage stochastic programming problem for the first two approaches. We solve SINSA with Benders' decomposition, which is a common method to solve two-stage stochastic programming problems. In the second approach, we apply the Lagrangian relaxation with Benders' decomposition to solve SINSA, in which we relax a budget constraint (2). We describe how the Lagrangian relaxation with Benders' decomposition can be applied as a search method for bicriteria programming problems. Lastly, SINSA is alternatively considered as a multistage stochastic programming problem for which its nested Benders' decomposition is demonstrated.

Unlike the traditional approaches that provide a single solution, we collect a set of nondominated solutions. Here, we refer to *nondominated solutions* as nurse staffs and assignments that are not dominated by any other staffs and assignments found; either they require less excess workload or less staffing cost than the other solutions found. Each solution represents staffs and assignments for nurses in an upcoming shift. We refer to the *efficient frontier* as a tradeoff curve between excess workload and staffing cost of the set of nondominated solutions found. The focus of this paper is to find many nondominated solutions to form the efficient frontier.

Algorithm 1 describes the algorithm for collecting the nondominated solutions. Let L be a list of nondominated solutions. Let $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj}, \overline{X})$ be a current solution obtained by solving SINSA with one of the three proposed approaches. We compare the current solution $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj}, \overline{X})$ with those in the list L. If the current solution $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj}, \overline{X})$ produces higher staffing cost or more excess workload than those in the list L, we discard the current solution $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj}, \overline{X})$. Otherwise, we add the current solution $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj}, \overline{X})$ into the list L. Then, we update the list of nondominated solutions by deleting solutions dominated by the current solution $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj}, \overline{X})$.

Algorithm 1 Collecting Nondominated Solutions Algorithm

Let L be a list of nondominated solutions.

Input: Let $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj})$ be a nurse staff, $\overline{X}$ be an assignment, and $Z_{(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj}, \overline{X})}$ be their objective function obtained by solving SINSA with one of the three algorithmic approaches.

if $\exists (Y_{nj}, Y^c_n, Y^r_{nj}, Y^o_{nj}, Y^a_{nj}, X) \in L$ in which $Z_{(Y_{nj}, Y^c_n, Y^r_{nj}, Y^o_{nj}, Y^a_{nj}, X)} < Z_{(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj}, \overline{X})}$ **then**

  Delete the current solution $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj}, \overline{X})$.

else

  Add the current solution $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj}, \overline{X})$ to a list L, $L \leftarrow L \cup (\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj}, \overline{X})$.

  for all $(Y_{nj}, Y^c_n, Y^r_{nj}, Y^o_{nj}, Y^a_{nj}, X) \in L$ **do**

    if $Z_{(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj}, \overline{X})} < Z_{(Y_{nj}, Y^c_n, Y^r_{nj}, Y^o_{nj}, Y^a_{nj}, X)}$ **then**

      Delete a solution $(Y_{nj}, Y^c_n, Y^r_{nj}, Y^o_{nj}, Y^a_{nj}, X)$ in a list L, $L \leftarrow L \backslash (Y_{nj}, Y^c_n, Y^r_{nj}, Y^o_{nj}, Y^a_{nj}, X)$.

    end if

  end for

end if

4.1 Benders' Decomposition

In this section, we consider SINSA as a two-stage stochastic integer programming problem. We solve SINSA using the L-shaped method based on Benders' decomposition with integer first-stage variables [11, 16]. The Benders' decomposition separates the original problem given by (1)-(18) into the master problem and the subproblems. The master problem determines scheduled nurses, PRN nurses, over-time nurses, and agency nurses working for a shift, and it assigns those nurses to patients with an objective of minimizing excess workload on nurses. Given the nurse staffing and their assignments, the recourse problems penalize the excess workload from the assignment. The subproblems decompose by the number of nurses and the number of scenarios into $|\mathbf{N}| \times |\xi|$ linear programming subproblems.

Let $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj})$ be a given nurse staffing, and let $\overline{X}$ be a given assignment. For each $t \in T$, let $\overline{g}^\xi_{tn} = \sum_{p\in P(n)} g^\xi_{tpn}\overline{X}_{pn}$, and let $\overline{d}^\xi_{tn} = \sum_{p\in P(n)} d^\xi_{tpn}\overline{X}_{pn}$. Our recourse subproblems are reduced to ones similar to those primal subproblems and dual subproblems of SPA described by Punnakitikashem et al. [63]. For each nurse $n \in \mathbf{N}$ and each scenario $\xi \in \Xi$, the *primal subproblem* is the following linear program (PS^ξ_n):

$$\min \sum_{\tau\in T}\sum_{i=1}^{k} \alpha_{\tau ni} A^\xi_{\tau ni} \tag{27}$$

$$\sum_{\tau=t}^{|T|} G^\xi_{t\tau n} = \overline{g}^\xi_{tn} \qquad \forall t \in T, \tag{28}$$

$$\sum_{i=1}^{k} A^\xi_{\tau ni} - \sum_{t=1}^{\tau} G^\xi_{t\tau n} = \overline{d}^\xi_{\tau n} \qquad \forall \tau \in T, \tag{29}$$

(A^ξ_n, G^ξ_n) satisfy (17), and (18).

Punnakitikashem et al. [63] demonstrated that (PS^ξ_n) is a network flow problem, which can be efficiently optimally solved by using a greedy algorithm (GAPS) [63]. GAPS determines the total workload variables $A^\xi_{\tau ni}$ and total indirect care variables $G^\xi_{t\tau n}$ performed by nurse n from the beginning of time period t until the end of the shift. This problem always has a feasible solution $(\tilde{A}, \tilde{G})$ that is $\tilde{G}^\xi_{ttn} = \overline{g}^\xi_{tn}$ and $\tilde{A}^\xi_{tnk} = \tilde{G}^\xi_{ttn} + \overline{d}^\xi_{tn}$ for all $t \in T$, and all other variables are zero. Let π^ξ_{tn}, $\mu^\xi_{\tau n}$, and $\rho^\xi_{\tau ni}$ be the dual variables associated with constraint sets (28) and (29) and the upper bounds in set (18), respectively. For each nurse $n \in \mathbf{N}$ and each scenario $\xi \in \Xi$, the dual subproblem (DS^ξ_n) is given by:

$$\max \sum_{t\in T}\left[\sum_{i=1}^{k}(m_{ti} - m_{t(i+1)})\rho^\xi_{tni}\right] + \overline{g}_t\pi^\xi_t + \overline{d}_t\mu^\xi_{tn} \tag{30}$$

$$\mu^\xi_{\tau n} - \rho^\xi_{\tau ni} \le \alpha_{\tau i} \qquad \forall \tau \in T, 1 \le i \le k, \tag{31}$$

$$\pi^\xi_{tn} \le \mu^\xi_{\tau n} \qquad \forall t, \tau \in T, t \le \tau, \tag{32}$$

$$\rho^\xi_{\tau ni} \ge 0 \qquad \forall \tau \in T, 1 \le i \le k, \tag{33}$$

$$\pi^\xi_{tn}, \mu^\xi_{\tau n} \text{ free} \qquad \forall t, \tau \in T. \tag{34}$$

The dual subproblem has a feasible solution in which all variables are zero. Consequently, the primal and dual subproblems have optimal solutions. Let DS denote the combination of all dual subproblems DS^ξ_n over all nurses and scenarios. Let Δ denote set of extreme points for the dual subproblem DS. The SINSA reformulation problem (SINSAR) can be written as follows:

$$\min \eta \tag{35}$$

subject to

$$\eta \ge \sum_{n\in\mathbf{N}}\sum_{\xi\in\Xi}\sum_{t\in T}\phi^\xi\left[\sum_{p\in P(n)}\left(\tilde{\pi}^\xi_{tn}g_{tpn} + \tilde{\mu}^\xi_{tn}d_{tpn}\right)X_{pn} + \sum_{i=1}^{k}(m_{tni} - m_{tn(i+1)})\tilde{\rho}^\xi_{tni}\right] \tag{36}$$

$$\forall (\tilde{\pi}, \tilde{\mu}, \tilde{\rho}) \in \Delta,$$

where $(Y_{nj}, Y^c_n, Y^r_{nj}, Y^o_{nj}, Y^a_{nj})$ satisfy (2)-(12),

X_{pn} satisfy (13) and (16).

Constraint set (36) associating with the extreme points of the optimal dual solutions is termed *optimality cuts*.

The L-shaped method is described as Algorithm 2. On each iteration, we consider a subset of dual extreme points $\overline{\Delta} \subseteq \Delta$, and let constraint set (36') be the subset of (36) over $\overline{\Delta}$. We solve a restricted master problem (2)-(13), (16), (35), and (36') to find a nurse staffing $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj})$, an assignment $\overline{X}$, and an anticipated objective value $\overline{\eta}$. Using the assignment $\overline{X}$, we solve the dual subproblems over all of the nurses and scenarios to obtain $(\tilde{\pi}, \tilde{\mu}, \tilde{\rho})$. Then, we collect all nondominated solutions in the algorithm by inputing a current nurse staffing $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj})$, an assignment $\overline{X}$ into Algorithm 1. If the anticipated objective value $\overline{\eta}$ is less than the objective value of the dual solution $(\tilde{\pi}, \tilde{\mu}, \tilde{\rho})$, then we add a Benders' optimality cut to (36'). Otherwise, the algorithm terminates and we obtain a set of nondominated solutions in the list L.

Algorithm 2 Stochastic Integrated Nurse Staffing and Assignment Benders' Decomposition Algorithm (SINSA-BD).

$\overline{\Delta} \leftarrow \emptyset$, $STOP \leftarrow FALSE$.
while $STOP = FALSE$ **do**
  Solve the restricted master problem (2)-(13), (16), (35), and (36') to obtain a nurse staffing $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj})$, an assignment $\overline{X}$ and an anticipated objective value $\overline{\eta}$. (On the first iteration, let $\overline{\eta} \leftarrow -\infty$, and let $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj})$ be a feasible nurse staffing, $\overline{X}$ be a feasible assignment.)
  for all $n \in \mathbf{N}$, $\xi \in \Xi$ **do**
    Solve the dual subproblem (DS^ξ_n) to obtain extreme point $(\tilde{\pi}^\xi_n, \tilde{\mu}^\xi_n, \tilde{\rho}^\xi_n)$.
  end for
  Keep a set of nondominated solutions by using Algorithm 1.
  if $\overline{\eta} < \sum_{p\in P} \sum_{n\in \mathbf{N}(p)} \sum_{\xi\in\Xi} \sum_{t\in T} \phi^\xi \left[\left(\tilde{\pi}^\xi_{tn} g_{tpn} + \tilde{\mu}^\xi_{tn} d_{tpn}\right) \overline{X}_{pn} + \sum_{i=1}^k (m_{tni} - m_{tn(i+1)}) \tilde{\rho}^\xi_{tni}\right]$ **then**
    $\overline{\Delta} \leftarrow \overline{\Delta} \cup \{(\tilde{\pi}, \tilde{\mu}, \tilde{\rho})\}$, where $(\tilde{\pi}, \tilde{\mu}, \tilde{\rho})$ is the combination of the vectors $(\tilde{\pi}^\xi_n, \tilde{\mu}^\xi_n, \tilde{\rho}^\xi_n)$.
  else
    $STOP \leftarrow TRUE$.
  end if
end while
return the list of nondominated solutions L.

4.2 Lagrangian Relaxation with Benders' Decomposition

In this section, we describe Lagrangian relaxation with Benders' decomposition for solving SINSA, and we use the subgradient method to determine the value of the Lagrange multiplier. Then, we describe how Lagrangian relaxation with Benders' decomposition can be viewed as a search method for bicriteria programming problems.

Lagrangian relaxation methods have been widely used to solve integer programming problems that contain sets of hard constraints and easy constraints. By dualizing hard constraints, we construct a Lagrangian problem that is relative easy to solve compared to the original problem. An optimal value of the Lagrangian problem is a lower bound on the optimal value of the original problem. Lagrangian relaxation efficiently solves integer programming problems since it provides better lower bounds than those from linear programming relaxation in a branch and bound algorithm. A review paper for Lagrangian relaxation for solving integer programming problems can be found in Fisher [33]. The *subgradient method* is a common technique that uses the Lagrangian dual problem, and it usually provides promising results. More information about the subgradient method is in Bertsekas [14] and Nemhauser and Wolsey [53]. Held et al. [35] described theoretical convergence properties and computational performance of subgradient optimization.

We propose to solve the SINSA by using Benders' decomposition, in which Lagrangian relaxation is employed to relax the budget constraint. We dualize the budget constraint (2) to the objective function and obtain the following *Lagrangian problem*:

$$\begin{aligned} L(\lambda) = \min \sum_{\xi\in\Xi}\sum_{n\in\mathbf{N}}\sum_{\tau\in T}\sum_{i=1}^{k}\phi^{\xi}\alpha_{\tau ni}A^{\xi}_{\tau ni} + \lambda\Bigg\{\sum_{j\in J}\sum_{n\in N(j)} c^s_{nj}Y_{nj} + \\ \sum_{n\in N} c^c_n Y^c_n + \sum_{j\in J}\sum_{n\in R(j)} c^r_{nj}Y^r_{nj} + \sum_{j\in J}\sum_{n\in O(j)} c^o_{nj}Y^o_{nj} + \\ \sum_{j\in J}\sum_{n\in A(j)} c^a_{nj}Y^a_{nj} - B\Bigg\} \end{aligned} \tag{37}$$

$$\text{subject to} \qquad (3)-(18),$$

where λ is a Lagrange multiplier. $L(\lambda)$ is a piecewise linear function. For any $\lambda \geq 0$, $L(\lambda)$ forms a lower bound on SINSA, as $\lambda(\sum_{j\in J}\sum_{n\in N(j)} c^s_{nj}Y_{nj} + \sum_{n\in N} c^c_n Y^c_n + \sum_{j\in J}\sum_{n\in R(j)} c^r_{nj}Y^r_{nj} + \sum_{j\in J}\sum_{n\in O(j)} c^o_{nj}Y^o_{nj} + \sum_{j\in J}\sum_{n\in A(j)} c^a_{nj}Y^a_{nj} - B) < 0$ for any feasible solutions to SINSA. The Lagrangian problem (3)-(18), (37) can be alternatively viewed as a bicriteria stochastic integer programming problem with objectives that minimize average excess workload on nurses and total nurse staffing cost. Given a nurse staffing $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj})$ and an assignment $\overline{X}$, the subproblems are separated by the total number of nurses and the total number of scenarios into $|\mathbf{N}| \times |\xi|$ linear programming subproblems. With the same fashion as the Benders' decomposition approach, for each nurse $n \in \mathbf{N}$ and each scenario $\xi \in \Xi$, the *primal subproblem* (PS^{ξ}_n) can be written by (17), (18), and (27)-(29), and the *dual subproblem* (DS^{ξ}_n) is given by (30)-(34).

We use the subgradient method to determine the Lagrange multiplier λ. The subgradient method for SINSA is described as Algorithm 3. Let r and α denote an iteration number and a step-size, respectively. For each iteration r, let θ_r be a parameter for the subgradient algorithm. On each iteration, we solve the Lagrangian problem by (3)-(18), (37) using the Benders' decomposition described in Algorithm 2 to obtain the nurse staffing $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj})$, the assignment $\overline{X}$. According to a given staffing and assignment, we update the step-size α and the Lagrange multiplier λ. If an absolute difference between the previous and current Lagrange multipliers are less than a positively small number ϵ, then we terminate the algorithm and obtain the list of nondominated solutions L. Otherwise, we update the parameter θ if the objective value does not improve. The iteration number is also updated. We repeat this iterative procedure until the termination criteria is met.

Typically, scheduling problems are difficult because of large solution search spaces. Many search methods have been developed for nurse scheduling problems, such as Tabu search, simulated annealing, genetic algorithm, etc. In this paper, we provided a novel search approach for a bicriteria stochastic integer program by using Lagrangian relaxation as a framework. A Lagrange multiplier acts as a penalty for violating the second objective and Benders' decomposition handles stochasticity in the model. According to our model, we penalize a staff that violates the budget for a shift. We find the nurse staffing and assignment that minimize both excess workload on nurses and budget violation in the Lagrangian problem. During the searching process, the Lagrangian relaxation with Benders' decomposition searches

Algorithm 3 Stochastic Integrated Nurse Staffing and Assignment Subgradient Algorithm (SINSA-SA).

Let r be an iteration number, α be step size, $\theta_0 \leftarrow 2$, initial Lagrange multiplier $\lambda^0 \geq 0$, $STOP \leftarrow FALSE$.

while $STOP = FALSE$ **do**

Solve the Lagrangian problem (3)-(18), (37) by the SINSA-BD described in Algorithm 2 to obtain the lower bound $\tilde{Z}_{LB}$.

$\alpha \leftarrow \theta(\tilde{Z}_{UB} - \tilde{Z}_{LB})/ (\sum_{j\in J}\sum_{n\in N(j)} c^s_{nj} Y_{nj} + \sum_{n\in N} c^c_n Y^c_n + \sum_{j\in J}\sum_{n\in R(j)} c^r_{nj} Y^r_{nj} + \sum_{j\in J}\sum_{n\in O(j)} c^o_{nj} Y^o_{nj} + \sum_{j\in J}\sum_{n\in A(j)} c^a_{nj} Y^a_{nj} - B)^2$.

$\lambda^{r+1} \leftarrow \max\{0, \lambda^r + \alpha(\sum_{j\in J}\sum_{n\in N(j)} c^s_{nj} Y_{nj} + \sum_{n\in N} c^c_n Y^c_n + \sum_{j\in J}\sum_{n\in R(j)} c^r_{nj} Y^r_{nj} + \sum_{j\in J}\sum_{n\in O(j)} c^o_{nj} Y^o_{nj} + \sum_{j\in J}\sum_{n\in A(j)} c^a_{nj} Y^a_{nj} - B)$.

if $|\lambda^{r+1} - \lambda^r| < \epsilon$ **then**

$STOP \leftarrow TRUE$

end if

if $|\tilde{Z}^{r+1}_{LB} - \tilde{Z}^r_{LB}| < \epsilon$ **then**

$\theta_{r+1} \leftarrow \theta_r/2$

else

$\theta_{r+1} \leftarrow \theta_r$

end if

$r \leftarrow r + 1$

end while

return the list of nondominated solutions L.

for solutions with different weights between average excess workload on nurses and budget violation.

4.3 Nested Benders' Decomposition

The nested Benders' decomposition method is a common solution method for multistage stochastic programming problems [16]. It is appropriate to use when the subproblems have block angular structure and involve further decomposition. In addition to multistage stochastic programs, the decomposition has been used successfully to solve the multistage convex programs [57]. More details about nested Benders' decomposition can be found in Birge [15] and Birge and Louveaux [16].

We develop a solution approach based on nested Benders' decomposition of SINSA. SINSA can be considered as a three-stage stochastic programming problem. The first-stage problem proposes a nurse staffing that determines nurses who work for the shift. Given a nurse staffing, the second-stage subproblem assigns nurses to a set of patients. Based upon an assignment, the third stage problems are decomposed into subproblems associated with each nurse at each scenario, and they evaluate assignments.

Given a nurse staffing $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj})$ and assignment $\overline{X}$, the subproblems can be reduced to ones similar to those from the two-stage stochastic program with Benders' decomposition. For each nurse $n \in N$ and each scenario $\xi \in \Xi$, the *third stage primal subproblem* $(PN^{3\xi}_n)$ is given by (17), (18), (27)-(29) and the *third stage dual subproblem* $(DN^{3\xi}_n)$ is given by (30)-(34). Let (DN^3) be the combination of all dual subproblems $(DN^{3\xi}_n)$ over all nurses and scenarios. Let Λ be the set of extreme points for the dual subproblem (DN^3). Note that the binary constraint (16) is relaxed to the upper bound constraint (44). Given a schedule of nurses, the *second-stage restricted master problem* (RMP^2) can be formulated as:

$$\min \eta^2 \tag{38}$$

subject to

$$\eta^2 \geq \sum_{n\in\mathbf{N}} \sum_{\xi\in\Xi} \sum_{t\in T} \phi^\xi \{ \sum_{p\in P(n)} (\tilde{\pi}^\xi_{tn} g_{tpn} + \tilde{\mu}^\xi_{tn} d_{tpn}) X_{pn} + \sum_{i=1}^{k} (m_{tni} - m_{tn(i+1)}) \tilde{\rho}^\xi_{tni} \} \quad \forall (\tilde{\pi}, \tilde{\mu}, \tilde{\rho}) \in \Lambda, \tag{39}$$

$$X_{pn} \leq \overline{Y}_{nj} \quad \forall n \in N(p), p \in P(j), j \in J, \tag{40}$$

$$X_{pn} \leq \overline{Y}^r_{nj} \quad \forall n \in R(p), p \in P(j), j \in J, \tag{41}$$

$$X_{pn} \leq \overline{Y}^o_{nj} \quad \forall n \in O(p), p \in P(j), j \in J, \tag{42}$$

$$X_{pn} \leq \overline{Y}^a_{nj} \quad \forall n \in A(p), p \in P(j), j \in J, \tag{43}$$

$$0 \leq X_{pn} \leq 1 \quad \forall p \in P, n \in \mathbf{N}(p), \tag{44}$$

where X_{pn} satisfy (13).

Constraints (39) are optimality cuts, which represent a successive linear approximation of the third stage problem. Let $(\psi_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})}, \sigma_{npj}, \beta_{npj}, \gamma_{npj}, \nu_{npj}, \chi_{pn}, \omega_p)$ be the dual variables associated with constraints (39)-(44), and (13) respectively. Let $(\psi^r_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})}, \sigma^r_{npj}, \beta^r_{npj}, \gamma^r_{npj}, \nu^r_{npj}, \chi^r_{pn}, \omega^r_p)$ be extreme rays of the dual polyhedron. The *second-stage* *dual problem* ($DRMP^2$) can be written as the following:

$$\begin{aligned}
\max \sum_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})\in\Lambda} &(\sum_{n\in\mathbf{N}}\sum_{\xi\in\Xi}\sum_{t\in T}\phi^{\xi}\sum_{i=1}^{k}(m_{tni}-m_{tn(i+1)})\tilde{\rho}^{\xi}_{tni})\psi_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})} \\
&-\sum_{j\in J}\sum_{p\in P(j)}\{\sum_{n\in N(p)}\overline{Y}_{nj}\sigma_{npj}+\sum_{n\in R(p)}\overline{Y}^{r}_{nj}\beta_{npj} \\
&+\sum_{n\in O(p)}\overline{Y}^{o}_{nj}\gamma_{npj}+\sum_{n\in A(p)}\overline{Y}^{a}_{nj}\nu_{npj}\}-\sum_{p\in P}\sum_{n\in\mathbf{N}(p)}\chi_{pn}+\sum_{p\in P}\omega_p
\end{aligned} \tag{45}$$

subject to

$$-\sum_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})\in\Lambda}\sum_{\xi\in\Xi}\sum_{t\in T}\phi^{\xi}(\tilde{\pi}^{\xi}_{tn}g_{tpn}+\tilde{\mu}^{\xi}_{tn}d_{tpn})\psi_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})}-\sigma_{npj}-\chi_{pn}+\omega_p\le 0$$

$$\forall j\in J, p\in P(j), n\in N(p), \tag{46}$$

$$-\sum_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})\in\Lambda}\sum_{\xi\in\Xi}\sum_{t\in T}\phi^{\xi}(\tilde{\pi}^{\xi}_{tn}g_{tpn}+\tilde{\mu}^{\xi}_{tn}d_{tpn})\psi_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})}-\beta_{npj}-\chi_{pn}+\omega_p\le 0$$

$$\forall j\in J, p\in P(j), n\in R(p), \tag{47}$$

$$-\sum_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})\in\Lambda}\sum_{\xi\in\Xi}\sum_{t\in T}\phi^{\xi}(\tilde{\pi}^{\xi}_{tn}g_{tpn}+\tilde{\mu}^{\xi}_{tn}d_{tpn})\psi_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})}-\gamma_{npj}-\chi_{pn}+\omega_p\le 0$$

$$\forall j\in J, p\in P(j), n\in O(p), \tag{48}$$

$$-\sum_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})\in\Lambda}\sum_{\xi\in\Xi}\sum_{t\in T}\phi^{\xi}(\tilde{\pi}^{\xi}_{tn}g_{tpn}+\tilde{\mu}^{\xi}_{tn}d_{tpn})\psi_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})}-\nu_{npj}-\chi_{pn}+\omega_p\le 0$$

$$\forall j\in J, p\in P(j), n\in A(p), \tag{49}$$

$$\sum_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})\in\Lambda}\psi_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})}=1, \tag{50}$$

$$\psi_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})}\ge 0 \qquad \forall(\tilde{\pi},\tilde{\mu},\tilde{\rho})\in\Lambda, \tag{51}$$

$$\sigma_{npj}\ge 0 \qquad \forall j\in J, p\in P(j), n\in N(p), \tag{52}$$

$$\beta_{npj}\ge 0 \qquad \forall j\in J, p\in P(j), n\in R(p), \tag{53}$$

$$\gamma_{npj}\ge 0 \qquad \forall j\in J, p\in P(j), n\in O(p), \tag{54}$$

$$\nu_{npj}\ge 0 \qquad \forall j\in J, p\in P(j), n\in A(p), \tag{55}$$

$$\chi_{pn}\ge 0 \qquad \forall p\in P, n\in\mathbf{N}(p), \tag{56}$$

$$\omega_p \quad \text{free} \qquad \forall p\in P. \tag{57}$$

Let Ψ (Γ) be the set of extreme points (extreme rays) of the second-stage dual problem (45)-(57). The *restricted* *master problem* (RMP^1) for the nested Benders' decomposition is reformulated as follows:

$$\min \eta^1 \tag{58}$$

subject to

$$\eta^1 \geq -\sum_{j\in J}\sum_{p\in P(j)}\{\sum_{n\in N(p)}\tilde{\sigma}_{npj}Y_{nj} + \sum_{n\in R(p)}\tilde{\beta}_{npj}Y^r_{nj}$$
$$+ \sum_{n\in O(p)}\tilde{\gamma}_{npj}Y^o_{nj} + \sum_{n\in A(p)}\tilde{\nu}_{npj}Y^a_{nj}\} + \overline{\psi} - \overline{\chi} + \overline{\varpi} \qquad \forall(\tilde{\psi},\tilde{\sigma},\tilde{\beta},\tilde{\gamma},\tilde{\nu},\tilde{\chi},\tilde{\omega}) \in \Psi, \tag{59}$$
$$-\sum_{j\in J}\sum_{p\in P(j)}\{\sum_{n\in N(p)}\tilde{\sigma}^r_{npj}Y_{nj} + \sum_{n\in R(p)}\tilde{\beta}^r_{npj}Y^r_{nj}$$
$$+ \sum_{n\in O(p)}\tilde{\gamma}^r_{npj}Y^o_{nj} + \sum_{n\in A(p)}\tilde{\nu}^r_{npj}Y^a_{nj}\} + \overline{\psi}^r - \overline{\chi}^r + \overline{\varpi}^r \leq 0 \qquad \forall(\tilde{\psi}^r,\tilde{\sigma}^r,\tilde{\beta}^r,\tilde{\gamma}^r,\tilde{\nu}^r,\tilde{\chi}^r,\tilde{\omega}^r) \in \Gamma, \tag{60}$$

where $(Y_{nj}, Y^c_n, Y^r_{nj}, Y^o_{nj}, Y^a_{nj})$ satisfy (2), (3), (8)-(12).

$$\text{where} \quad \overline{\psi} = \sum_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})\in\Lambda}(\sum_{n\in\mathbf{N}}\sum_{\xi\in\Xi}\sum_{t\in T}\phi^{\xi}\sum_{i=1}^{k}(m_{tni} - m_{tn(i+1)})\tilde{\rho}^{\xi}_{tni})\tilde{\psi}_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})},$$
$$\overline{\varpi} = \sum_{p\in P}\tilde{\omega}_p,$$
$$\overline{\chi} = \sum_{p\in P}\sum_{n\in\mathbf{N}(p)}\tilde{\chi}_{pn},$$
$$\overline{\psi}^r = \sum_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})\in\Lambda}(\sum_{n\in\mathbf{N}}\sum_{\xi\in\Xi}\sum_{t\in T}\phi^{\xi}\sum_{i=1}^{k}(m_{tni} - m_{tn(i+1)})\tilde{\rho}^{\xi}_{tni})\tilde{\psi}^r_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})},$$
$$\overline{\varpi}^r = \sum_{p\in P}\tilde{\omega}^r_p,$$
$$\overline{\chi}^r = \sum_{p\in P}\sum_{n\in\mathbf{N}(p)}\tilde{\chi}^r_{pn}$$

Constraints (59) are the optimality cuts passing information from the second-stage dual problem $(DRMP^2)$ to the restricted master problem (RMP^1). When the second-stage problem (RMP^2) is infeasible, the *feasibility cuts* in constraints (60) are added to the restricted master problem (RMP^1) to induce a feasible solution.

Figure 1 illustrates the flow chart of nested Benders' decomposition method for SINSA. Note that our third stage subproblems $(PN^{3\xi}_n)$ for all of the nurses and scenarios are always feasible. The nested Benders' decomposition algorithm for the three-stage integrated nurse staffing and assignment problem (SINSA-NBD) is described as Algorithm 4.

The nested L-shaped method proceeds as follows. Let $SolveRMP1$ denote a boolean variable which is $true$ when we need to solve the restricted master problem (RMP^1). Let $CheckRMP1$ and $CheckRMP2$ be boolean variables. If $CheckRMP1$ is $true$, then the current restricted master problem (RMP^1) is checked whether it is optimal with respect to the first-stage optimality cut. If $CheckRMP2$ is $true$, then the current second-stage restricted master problem (RMP^2) is checked whether it is optimal with respect to the second-stage optimality cut. On each iteration, we consider a subset of dual extreme points of (DS^3) $\overline{\Lambda} \subseteq \Lambda$, and let constraint set (39') be the subset of

(39) over $\overline{\Lambda}$. We consider a subset of dual extreme points of $(DRMP^2)$ $\overline{\Psi} \subseteq \Psi$, and let constraint set (59') be the subset of (59) over $\overline{\Psi}$. We also consider a subset of dual extreme rays of $(DRMP^2)$ $\overline{\Gamma} \subseteq \Gamma$, and let constraint set (60') be the subset of (60) over $\overline{\Gamma}$. We solve a restricted master problem (2), (3), (8)-(12), (58), (59'), and (60') to find a nurse staffing $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj})$. Give a nurse staffing $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj})$, we solve the second-stage restricted master problem (13), (38), (39'), (40)-(44) and obtain a nurse assignment $\overline{X}$ and anticipated penalty $\overline{\eta}^2$. If a current assignment is infeasible, then we add a set of feasibility cuts (60) to the restricted master problem (RMP^1) and update the variable $solveRMP1$ to resolve the restricted master problem (RMP^1) until a feasible assignment is obtained. Otherwise, we update the variables $CheckRMP1$ and $CheckRMP2$ to check whether the restricted master problem (RMP^1) and the second-stage restricted master problem (RMP^2) are optimal with respect to the optimality cuts. If the variable $CheckRMP2$ is true, we solve the dual subproblems (DS^3) over all of the nurses $n \in \mathbf{N}$ and scenarios $\xi \in \Xi$ to obtain the optimal dual solutions $(\tilde{\pi}, \tilde{\mu}, \tilde{\rho})$. If constraints (39) with these dual solutions $(\tilde{\pi}, \tilde{\mu}, \tilde{\rho})$ are violated by the current assignment $\overline{X}$ and anticipated objective value $\overline{\eta}^2$, then we add a Benders' optimality cut to (39') and update the variable $CheckRMP2$ to resolve and check the second-stage restricted master problem (RMP^2). Otherwise, we adjust the variable $CheckRMP1$ to check an optimality cut for the restricted master problem (RMP^1). If the $CheckRMP1$ is $true$, then we perform the following. We collect all nondominated solutions in the algorithm by inputing a current nurse staffing $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj})$, an assignment $\overline{X}$ into Algorithm 1. If the anticipated objective value $\overline{\eta}^1$ is less than the objective value of the dual solution $(\tilde{\psi}_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})}, \tilde{\sigma}_{npj}, \tilde{\beta}_{npj}, \tilde{\gamma}_{npj}, \tilde{\nu}_{npj}, \tilde{\chi}_{pn}, \tilde{\omega}_p)$, then we add a Benders' optimality cut to (59') and all boolean variables are adjusted to resolve and check both restricted master problem (RMP^1) and second-stage restricted master problem (RMP^2). Then, another iteration is performed. Otherwise, the algorithm terminates and we obtain a set of nondominated solutions in the list L.

5 Computational Study

We report a computational study on integrated nurse staffing and assignment in this section. We tested the three solution approaches on four problem instances generated from data from a Northeast Texas hospital. The problem instances and parameters for our model are described in Section 5.1. Because of the complexity of the model, these problem instances cannot be solved optimally within 30 minutes. However, finding the optimal solutions may be meaningless since the nurse supervisor wants to quickly obtain a high-quality schedule and assignment that satisfy all requirements. Accordingly, the focus of the computational study is to find good nondominated solutions within a 30-minute time limit. In Section 5.2, we select the appropriate parameters for the solution approaches, and then the algorithmic efficiencies of the three approaches are compared. In Section 5.2.5, we study the effects of requiring nurses to work on their primary work units versus allowing nurses to float to other units. Results are stated in the same section.

5.1 Problem Instances

The Northeast Texas hospital provided encrypted data from two medical-surgical units, namely, Med-Surg1 and Med-Surg2, for this study, and the data was from March 2004-December 2004. We obtained encrypted patient data including a patient's primary diagnosis, room location, admission date, discharge date, and units in which the patient stayed. In

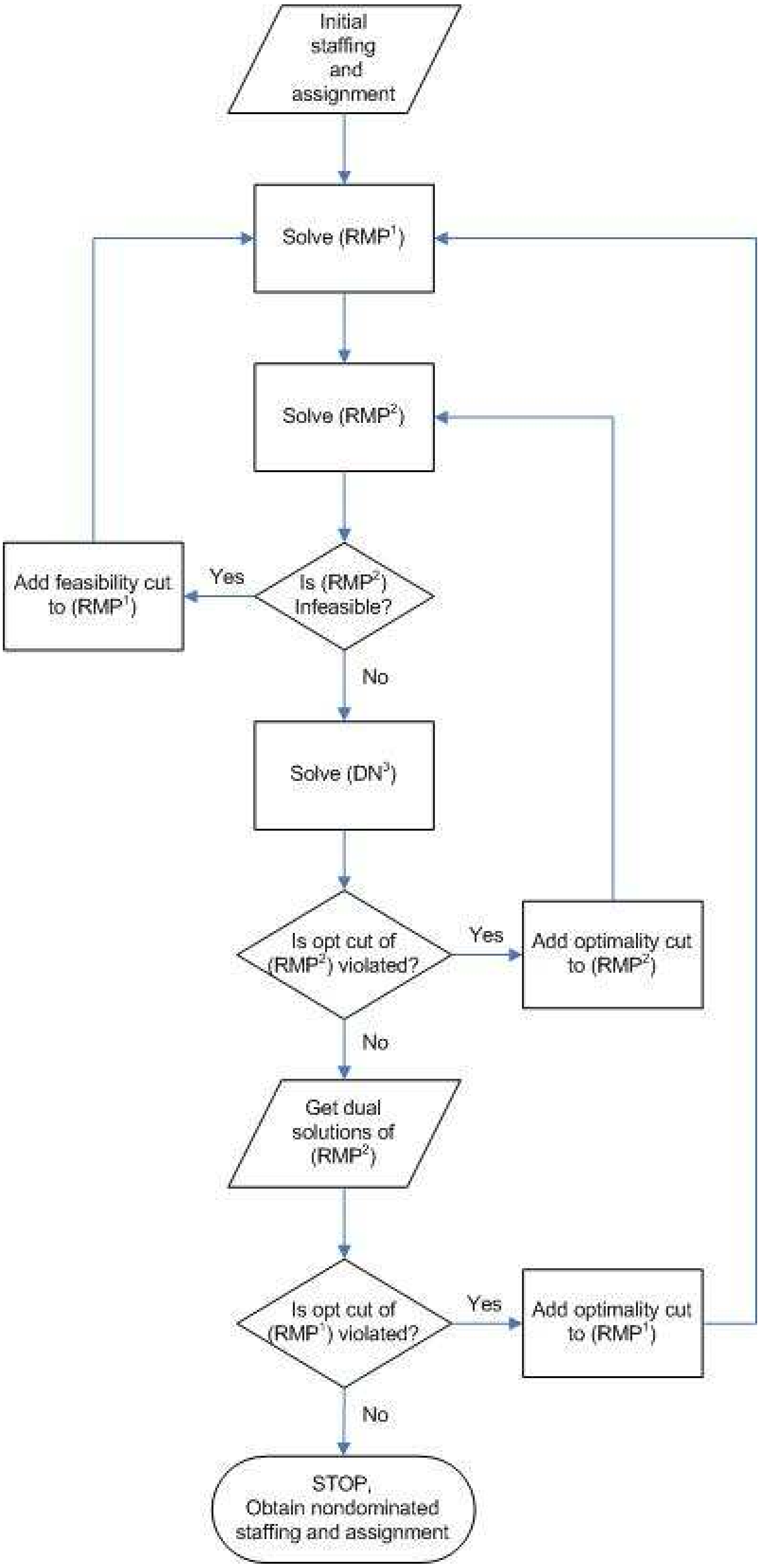

Figure 1: *The nested Benders' decomposition algorithm for the three-stage integrated nurse staffing and assignment problem flow chart*

Algorithm 4 Nested Benders' Decomposition Algorithm for the Three-Stage Integrated Nurse Staffing and Assignment Problem (SINSA-NBD).

Initialization: $\overline{\Theta} \leftarrow \emptyset$, $\overline{\Lambda} \leftarrow \emptyset$, $\overline{\Psi} \leftarrow \emptyset$, $\overline{\Gamma} \leftarrow \emptyset$, $solveRMP1 \leftarrow TRUE$, $CheckRMP1 \leftarrow TRUE$, $CheckRMP2 \leftarrow TRUE$.

while $solveRMP1 = TRUE || CheckRMP1 = TRUE || CheckRMP2 = TRUE$ **do**

  if $solveRMP1 = TRUE$ **then**

    Solve the restricted master problem (RMP^1) to obtain a nurse staffing $(\overline{Y}_{nj}, \overline{Y}^c_n, \overline{Y}^r_{nj}, \overline{Y}^o_{nj}, \overline{Y}^a_{nj})$ and an anticipated objective value $\overline{\eta}^1$. (On the first iteration, let $\overline{\eta}^1 \leftarrow -\infty$).

  end if

  Solve the second-stage restricted master problem (RMP^2) to obtain a nurse assignment $\overline{X}$ and an anticipated objective value $\overline{\eta}^2$. (On the first iteration, let $\overline{\eta}^2 \leftarrow -\infty$).

  if the second-stage restricted master problem (RMP^2) is infeasible **then**

    $\overline{\Gamma} \leftarrow \overline{\Gamma} \cup \left\{(\tilde{\psi}^r, \tilde{\sigma}^r, \tilde{\beta}^r, \tilde{\gamma}^r, \tilde{\nu}^r, \tilde{\chi}^r, \tilde{\omega}^r)\right\}$, where $(\tilde{\psi}^r, \tilde{\sigma}^r, \tilde{\beta}^r, \tilde{\gamma}^r, \tilde{\nu}^r, \tilde{\chi}^r, \tilde{\omega}^r)$ is the combination of the extreme rays $(\tilde{\psi}^r_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})}, \tilde{\sigma}^r_{npj}, \tilde{\beta}^r_{npj}, \tilde{\gamma}^r_{npj}, \tilde{\nu}^r_{npj}, \tilde{\chi}^r_{pn}, \tilde{\omega}^r_p)$.

    $solveRMP1 \leftarrow TRUE$, $CheckRMP1 \leftarrow FALSE$, and $CheckRMP2 \leftarrow FALSE$.

  else

    $solveRMP1 \leftarrow FALSE$, $CheckRMP1 \leftarrow TRUE$, and $CheckRMP2 \leftarrow TRUE$.

  end if

  if $CheckRMP2 = TRUE$ **then**

    for all $n \in \mathbf{N}$, $\xi \in \Xi$ **do**

      Solve the third stage dual subproblem ($DN^{3\xi}_n$) to obtain the extreme points $(\tilde{\pi}^\xi_n, \tilde{\mu}^\xi_n, \tilde{\rho}^\xi_n)$.

    end for

    if $\overline{\eta}^2 < \sum_{p \in P} \sum_{n \in \mathbf{N}(p)} \sum_{\xi \in \Xi} \sum_{t \in T} \phi^\xi \left[\left(\tilde{\pi}^\xi_{tn} g_{tpn} + \tilde{\mu}^\xi_{tn} d_{tpn}\right) \overline{X}_{pn} + \sum_{i=1}^k (m_{tni} - m_{tn(i+1)}) \tilde{\rho}^\xi_{tni}\right]$ **then**

      $\overline{\Lambda} \leftarrow \overline{\Lambda} \cup \{(\tilde{\pi}, \tilde{\mu}, \tilde{\rho})\}$, where $(\tilde{\pi}, \tilde{\mu}, \tilde{\rho})$ is the combination of the vectors $(\tilde{\pi}^\xi_n, \tilde{\mu}^\xi_n, \tilde{\rho}^\xi_n)$.

      $solveRMP1 \leftarrow FALSE$, $CheckRMP1 \leftarrow FALSE$, and $CheckRMP2 \leftarrow TRUE$.

    else

      $solveRMP1 \leftarrow TRUE$, $CheckRMP1 \leftarrow TRUE$, and $CheckRMP2 \leftarrow FALSE$.

    end if

  end if

  if $CheckRMP1 = TRUE$ **then**

    Keep a set of nondominated solutions by using Algorithm 1.

    if $\overline{\eta}^1 < \sum_{(\tilde{\pi},\tilde{\mu},\tilde{\rho}) \in \Lambda} (\sum_{n \in \mathbf{N}} \sum_{\xi \in \Xi} \sum_{t \in T} \phi^\xi \sum_{i=1}^k (m_{tni} - m_{tn(i+1)}) \tilde{\rho}^\xi_{tni}) \tilde{\psi}_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})} - \sum_{j \in J} \sum_{p \in P(j)} \left\{\sum_{n \in N(p)} \overline{Y}_{nj} \tilde{\sigma}_{npj} + \sum_{n \in R(p)} \overline{Y}^r_{nj} \tilde{\beta}_{npj} + \sum_{n \in O(p)} \overline{Y}^o_{nj} \tilde{\gamma}_{npj} + \sum_{n \in A(p)} \overline{Y}^a_{nj} \tilde{\nu}_{npj}\right\} + \sum_{p \in P} \tilde{\omega}_p - \sum_{p \in P} \sum_{n \in \mathbf{N}(p)} \tilde{\chi}_{pn}$ **then**

      $\overline{\Psi} \leftarrow \overline{\Psi} \cup \left\{(\tilde{\psi}, \tilde{\sigma}, \tilde{\beta}, \tilde{\gamma}, \tilde{\nu}, \tilde{\chi}, \tilde{\omega})\right\}$, where $(\tilde{\psi}, \tilde{\sigma}, \tilde{\beta}, \tilde{\gamma}, \tilde{\nu}, \tilde{\chi}, \tilde{\omega})$ is the combination of the vectors $(\tilde{\psi}_{(\tilde{\pi},\tilde{\mu},\tilde{\rho})}, \tilde{\sigma}_{npj}, \tilde{\beta}_{npj}, \tilde{\gamma}_{npj}, \tilde{\nu}_{npj}, \tilde{\chi}_{pn}, \tilde{\omega}_p)$.

      $solveRMP1 \leftarrow TRUE$, $CheckRMP1 \leftarrow TRUE$, and $CheckRMP2 \leftarrow TRUE$.

    else

      $solveRMP1 \leftarrow FALSE$, $CheckRMP1 \leftarrow FALSE$, and $CheckRMP2 \leftarrow FALSE$.

    end if

  end if

end while

return the list of nondominated solutions L.

Instance	Shift	Med-Surg1 unit		Med-Surg2 unit	
		No. of patients	No. of nurses	No. of patients	No. of nurses
1	Day	23	2-1	23	2-1
2	Day	18	4-0	18	4-0
3	Evening	18	2-1	18	2-1
4	Night	13	1-1	13	1-1

Table 1: *Instances generated from the Northeast Texas hospital data over ten months.*

Nurse Type	$/Shift	Total no. of nurses
Regular Nurse	160	6-8-6-4
Overtime Nurse	240	4
PRN Nurse	256	4
Agency Nurse	320	4

Table 2: *Salary of different types of nurses.*

addition to patient data, the Northeast Texas hospital gave nurse data as well. Each nurse at the hospital wears a badge that locates nurses in the hospital unit, so that a charge nurse can reach a nurse immediately when her patient calls the nurses' station. Given that the Northeast Texas hospital has a nurse locator device with RFID technology, they can track the nurses' locations from their badges and collect location data for nurses over months.

Problem instances were generated based upon encrypted patient data and nurse data. We used the Med-Surg1 unit instances created by Punnakitkashem et al. [63]. The Med-Surg2 unit instances were generated with the same fashion as those from Med-Surg1. We sampled a random set of patients from an empirical distribution of patients with similar diagnoses and patient rooms. Then, the number of nurses and type of nurses required for a shift was determined by a census matrix from a medical-surgical unit. We randomly generated 500 and 5000 scenarios for Ξ. The probability of each scenario is equally likely. We estimated patient admissions and discharges to the units as Poisson processes with mean equal to the number of patients in a shift divided by the average length of stay. Our data indicates that the average length of stay of patients in Med-Surg1 and Med-Surg2 unit were 2.725 and 1.936 days per patient, respectively. Besides patient information, individual nurse skills of regular nurses were also taken into consideration. However, the Northeast Texas hospital does not have PRN nurses, overtime nurses, and agency nurses data, therefore we assume they are identical. Table 1 represents characteristics of the four problem instances from two medical-surgical units. The column labeled "Instance" is the random instance, "Shift" is the time of the shift, and "No. of Pat" is the number of patients in the instance. The column labeled "No. of Nurses" is in the format of *a-b*, where *a* and *b* represent the number of registered nurses and licensed vocational nurses on duty, respectively. Table 2 displays salary for each type of nurses. The column labeled "Nurse Type" represents types of nurses. The column labeled "$/shift" displays the salary of a nurse per shift, and "Total No. of Nurses" shows the total number of nurses. The row labeled "Regular Nurse" is in the format of *c-d-e-f*, where *c*, *d*, *e*, and *f* represent the number of regular nurses in Instances 1-4, respectively.

5.2 Computational Results

In this section, we present computational results based upon instances created from real data from the Northeast Texas hospital described in Section 5.1. We begin with determining appropriate parameters for the Lagrangian relaxation with Benders' decomposition and the nested Benders' decomposition methods in Section 5.2.1. In Section 5.2.4, we compare expected excess workload and staffing cost from three different solution methods. The tradeoff between average excess workload and staffing cost are shown. Finally, we perform a computational comparison between two policies, which are with and without a reasonable assumption that nurses should be restricted to work on their primary work units in Section 5.2.5.

We implemented three different solution approaches in the C programming language on a Dell Precision Workstation with dual 3.06-Gz Intel Xeon processors using CPLEX 9.1 callable library. We solved SINSA with Benders' decomposition, Lagrangian relaxation with Benders' decomposition, and nested Benders' decomposition approaches, denoted as SINSA-BD, SINSA-LRBD, SINSA-NBD, respectively. We solved the mean value problem, which is a deterministic integer programming replaced direct care and indirect care random variables with their mean, for less than one minute to find an initial solution for all methods. Then, the problem was solved by each solution method for the remaining time. Punnakitikashem et at. [63] demonstrated that the primal subproblems (PS_n^{ξ}) are network flow problems, which can be efficiently solved by using the greedy algorithm (GAPS) [63]. They reported that the recourse function converged by evaluating it with 5000 scenarios. They showed that optimizing the problem with 500 scenarios and evaluating solutions by 5000 scenarios with GAPS provided the best results, which are minimal excess workload for nurses. Therefore, we optimize SINSA-BD, SINSA-LRBD, and SINSA-NBD with 500 scenarios and evaluate the recourse subproblems by using the greedy algorithm (GAPS) with 5000 scenarios to obtain the excess workload of each staff and assignment. In addition, we solved SINSA-BD and SINSA-NBD with a budget of \$3000 for the first 10 minutes, \$2000 for another 10 minutes, and \$1000 for the last 10 minutes. We solved SINSA-LRBD with a starting budget of \$2000.

According to the staffing policy from the Northeast Texas hospital, the staffing happens per unit with different managers. Med-Surg1 and Med-Surg2 nurses are assigned to their primary work units. Nevertheless, two medical-surgical units use the same nurse pool of PRN nurses, overtime nurses, and agency nurses. Therefore, we apply the following assumption:

Assumption 4: Scheduled nurses must be assigned to their primary work units. PRN nurses, overtime nurses, and agency nurses can be staffed to any unit needing help.

5.2.1 Parameter Tuning

In this section, we determine appropriate parameters for solving SINSA-LRBD and SINSA-NBD.

5.2.2 Parameters for SINSA-LRBD

We find proper parameters for the SINSA-LRBD approach in this section. Table 3 depicts parameters for SINSA-SA, which are initial step-size α, initial Lagrange multiplier λ, parameter θ, small positive number ϵ, and the iteration limit.

Parameter	Value
Initial step size α	2.0
Initial Lagrange multiplier λ	0.0
Initial LRBD parameter θ	2
ϵ	0.00005
Iteration number	2000

Table 3: *Parameters for the SINSA-LRBD approach.*

The termination criteria for SINSA-SA algorithm are the following:

1. The Lagrange multiplier converges within the small positive number ϵ. The difference between the previous and the current Lagrange multiplier is less than the small positive number ϵ.

2. The time limit is met. We use a 30-minute time limit in our computational results.

3. The iteration number limit is reached. The limitation for solving SINSA-SA is 2000 iterations.

We solved SINSA-LRBD by employing the subgradient algorithm for stochastic integrated nurse staffing and assignment (SINSA-SA) in Algorithm 3. According to SINSA-SA, we solve SINSA-BD described in Algorithm 2 and update the Lagrange multiplier and the step-size. Then, we perform another iteration until the termination criteria is met. One problem with SINSA-SA is that we cannot optimally solve SINSA-BD within 30 minutes, therefore not enough solutions are generated to form the efficient frontier. One way to overcome this problem is to set time limit for solving SINSA-BD. Accordingly, we examined time duration for solving SINSA-BD embedded in SINSA-SA that yields the best results. We solved SINSA-BD for 30, 60, 120, and 300 seconds, and then updated the step-size, Lagrange multiplier, and parameter θ. Tables 13-16 in an appendix show excess workload and staffing cost of solving SINSA-BD with different time limits embedded in SINSA-SA for all four instances, respectively. Figure 2 displays the efficient frontiers of solving SINSA-BD with different time limits embedded in SINSA-SA, and they indicate that solving SINSA-BD with 300 seconds within SINSA-SA provided the best results. Thus, we solved SINSA-BD for 300 seconds before updating the subgradient parameters in SINSA-SA in the remainder of this computational study.

5.2.3 SINSA-NBD Algorithm Enhancement

SINSA-NBD confronted the same situation as SINSA-LRBD that not many solutions were produced when we tried to solve the restricted master problem (RMP^1) optimally within 30 minutes. Hence, we enforced a different time limit and solved the restricted master problem (RMP^1) in SINSA-NBD described in Algorithm 1. Only one nondominated solution was obtained leading us to incorporate the following *minimum nurses constraint* to the restricted master problem (RMP^1) along with time limit to enhance the algorithmic performance.

$$\sum_{j \in J} \left\{ \sum_{n \in N(j)} Y_{nj} + \sum_{n \in R(j)} Y^r_{nj} + \sum_{n \in O(j)} Y^o_{nj} + \sum_{n \in A(j)} Y^a_{nj} \right\} \geq lb + \lfloor rand * (ub - lb + 1) \rfloor . \tag{61}$$

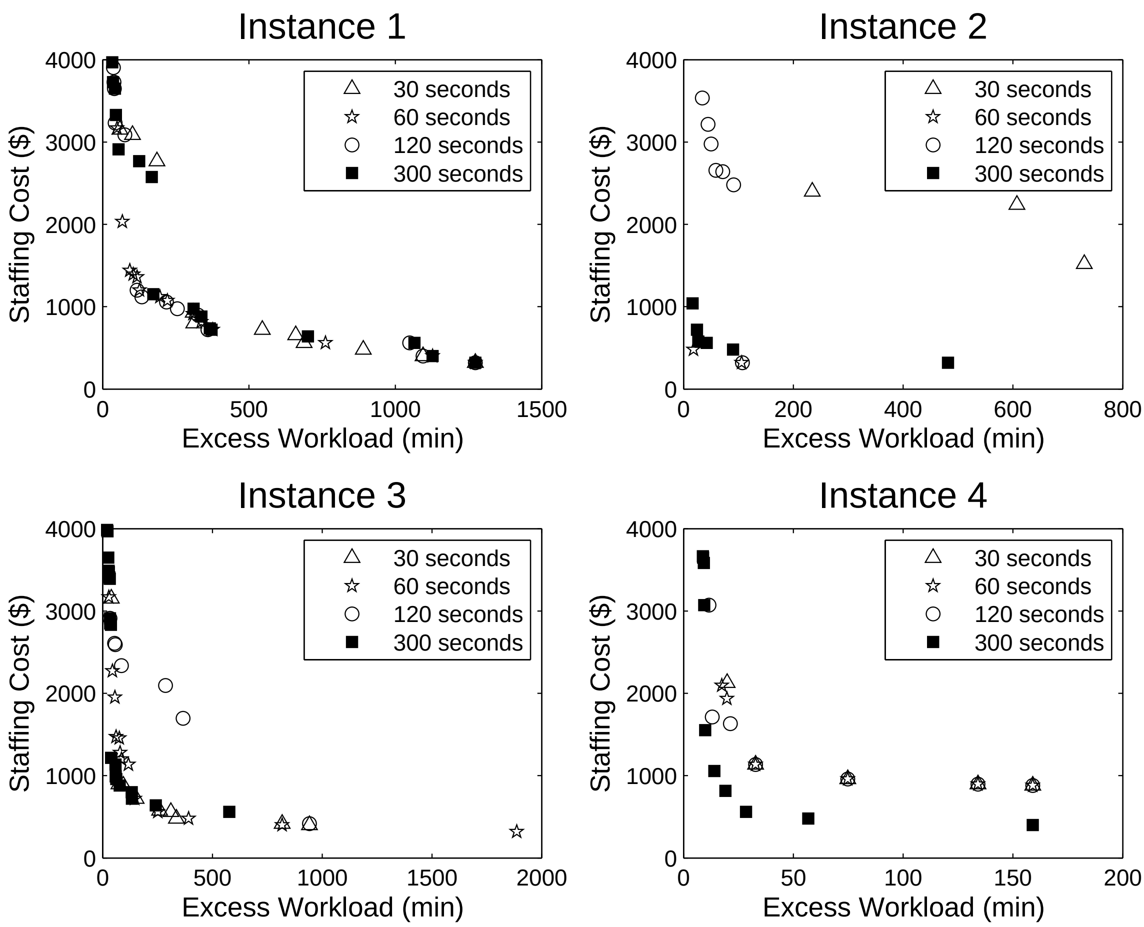

Figure 2: *Selecting time limit (in seconds) for the SINSA-LRBD approach*

where

$$rand = \text{random number}, \tag{62}$$

$$lb = 1, \tag{63}$$

$$ub = \left\lfloor \frac{|\mathbf{N}|}{|J|} \right\rfloor - \{|R| + |O| + |A|\} / |J|. \tag{64}$$

The minimum nurses constraint (61) randomly changes the number of nurses required to be staffed for a shift. As the algorithm was forced to explore many nurse staffs with different numbers of nurses, more quality nondominated solutions were generated.

With constraint (61), we investigated the time limit for solving the restricted master problem (RMP^1) within SINSA-NBD that gave the best staffing and assignments. We solved the restricted master problem (RMP^1) with 30, 60, 120, and 300 seconds. Tables in an appendix (17)-(20) display excess workload and staffing cost for solving the restricted master problem (RMP^1) with different time limits in SINSA-NBD for all four instances, respectively, and Figure 3 displays their efficient frontiers. Results illustrate that solving the restricted master problem (RMP^1) with 300 seconds gave minimum excess workload and staffing cost, therefore we included the minimum nurses constraint (61) and solved the restricted master problem (RMP^1) with 300 seconds in SINSA-NBD in the remainder of this computational study.

5.2.4 Algorithmic Approaches Comparison

In this section, we evaluated the algorithmic performance of three solution methods, which were SINSA-BD, SINSA-LRBD, and SINSA-NBD. We optimized SINSA with 500 scenarios with all three methods and evaluated the staffing and assignment decision with 5000 scenarios. Tables 4-7 summarize the average excess workload in minutes, the staffing costs, and the total number of nurses scheduled for a shift with different solution methods for all four instances, respectively. The breakdown to the number of each type of nurses, i.e., PRN nurses, overtime nurses, and agency nurses, scheduled to work for a shift is displayed in Table 21-24 in the Appendix. Note that solutions shown in Tables 4-7 and Tables 21-24 are the nondominated solutions. Each approach generated more solutions than those shown in tables, and we display the total number of solutions in Table 8. Figure 4 shows the efficient frontiers comparing average excess workload and staffing cost from solving SINSA with three different approaches.

In general, SINSA-BD is a good choice when the nurse supervisor has a certain amount of budget, because it provides several good solutions around the budget. SINSA-LRBD performs well with a flexible budget, as it generates more nondominated solutions on the efficient frontiers. We observed that many nondominated solutions showed up around \$2000 with the SINSA-BD approach because nurse staffs that cost around \$2000 have almost no excess workload.

Results also suggested that simultaneously staffing and assigning nurses provided better solutions than sequentially considering them. Moreover, the nurse staffing and assignments found by these methods can be used in a nurse staffing decision supporting system for the nurse supervisor. Not only can the nurse supervisor make a revised nurse schedule based on the tradeoff between staffing cost and excess workload on nurses, but she also obtains assignments of nurses to patients.

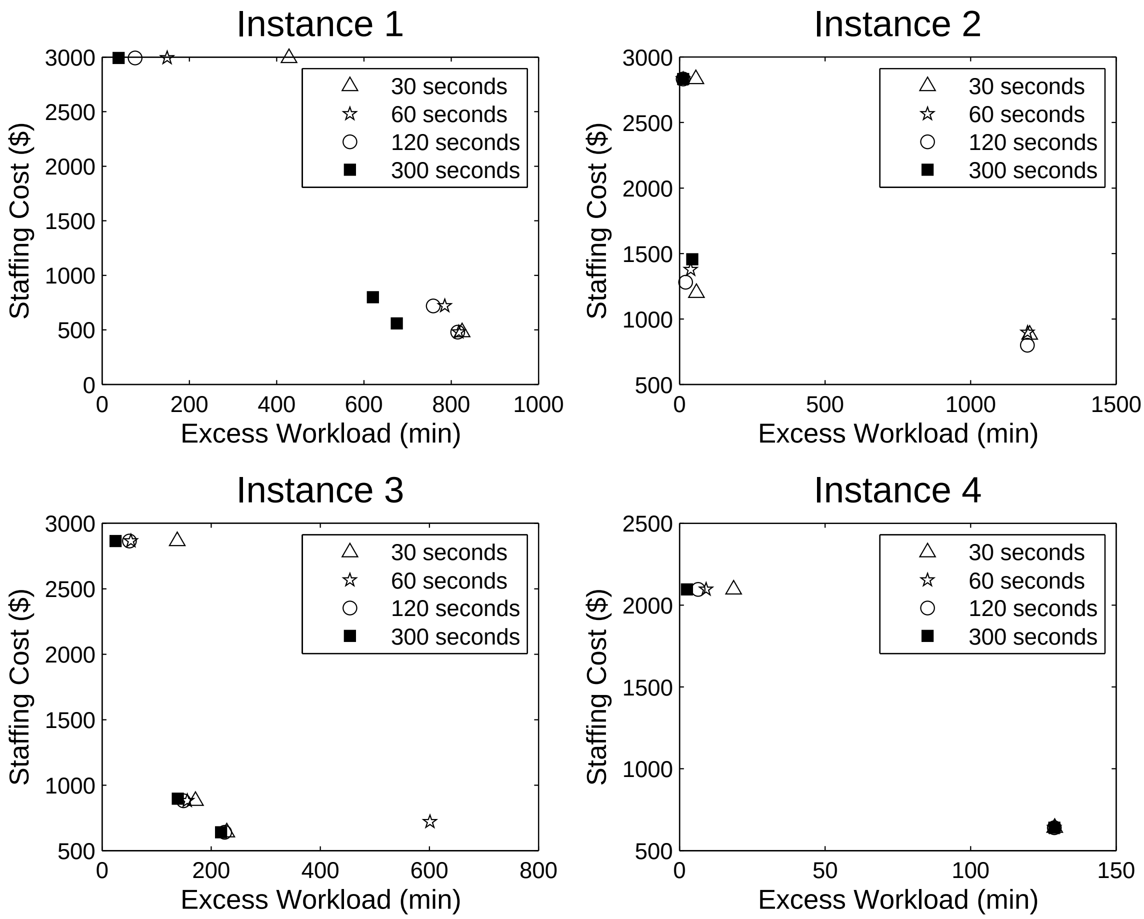

Figure 3: *Selecting time limit (in seconds) for the SINSA-NBD approach.*

SINSA-BD			SINSA-LRBD			SINSA-NBD		
Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses
32.68	2912	13	32.83	3968	17	37.89	2992	13
35.37	2784	12	35.48	3728	16	620.64	800	4
56.00	1936	9	42.12	3648	16	675.24	560	3
59.64	1872	9	45.09	3328	15			
68.07	1856	9	54.60	2912	13			
261.57	976	5	125.21	2768	13			
284.13	960	5	168.12	2576	12			
337.62	880	4	172.93	1152	6			
1132.64	720	3	311.06	976	5			
1680.28	576	2	337.68	880	4			
			366.12	736	4			
			372.15	720	4			
			701.54	640	4			
			1065.57	560	3			
			1127.59	400	2			
			1273.56	320	2			

Table 4: *Instance 1 results comparing average excess workload and staffing cost from solving SINSA with three different approaches.*

A solution selected by the nurse supervisor has a direct effect on nurses and patients for the entire shift. Rather than theoractically selecting a solution which is closest to the origin, we suggest the nurse supervisor to select the solution that provides the least amount of expected excess workload while satisfying a daily budget.

5.2.5 Working in Primary Work Units only vs. Floating to Other Units

In this section, we performed a computational experiment to evaluate two staffing policies:

Policy 1: Regular nurses are restricted to work in their primary work units only.

Policy 2: Regular nurses are allowed to float to other units in which they are qualified.

In both policies, PRN nurses, overtime nurses, and agency nurses can be staffed to any units in which they are qualified to work. Given that SINSA-LRBD provided reasonable results among three approaches, we employed SINSA-LRBD to compare these two policies. Tables 9-12 show the average excess workload and staffing cost of the nondominated schedules and assignments from the two policies for all four instances. Figure 5 illustrates the efficient frontiers from the two policies. Results indicated that working only in primary work units provided less excess workload with less staffing cost in Instances 2 and 3. The explanation is that the solution space was reduced by enforcing regular nurses to

SINSA-BD			SINSA-LRBD			SINSA-NBD		
Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses
14.14	1936	9	16.51	1040	5	12.86	2832	13
24.26	1872	9	24.35	720	4	44.22	1456	7
33.39	1760	9	26.92	576	3			
43.87	976	5	42.25	560	3			
60.48	880	5	90.17	480	3			
707.45	800	3	481.57	320	2			
750.17	416	2						

Table 5: *Instance 2 results comparing average excess workload and staffing cost from solving SINSA with three different approaches.*

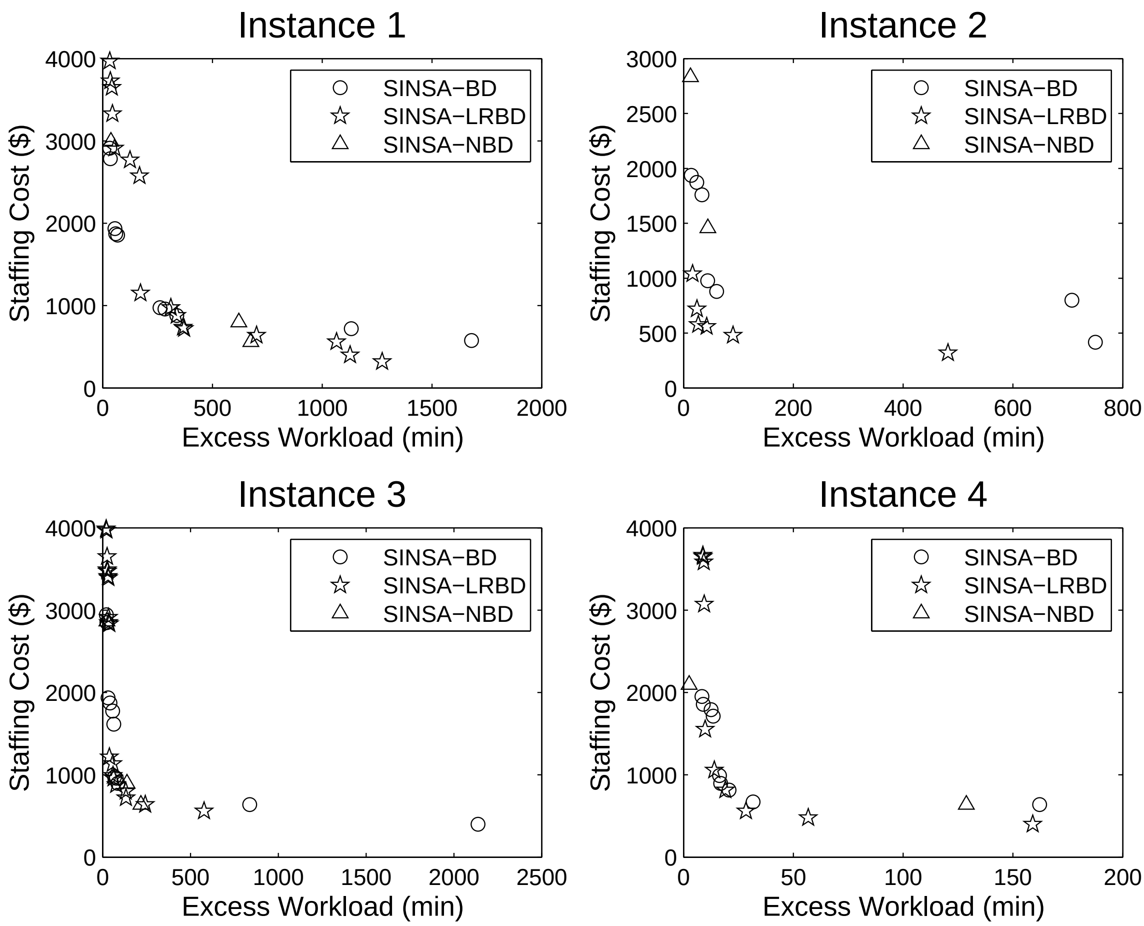

Figure 4: *Efficient frontiers comparing average excess workload and staffing cost from solving SINSA with three different approaches for all four instances.*

SINSA-BD			SINSA-LRBD			SINSA-NBD		
Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses
21.71	2944	13	20.42	3984	17	24.81	2864	11
24.60	2848	12	22.57	3968	17	138.96	896	4
30.53	1936	9	25.22	3648	16	218.17	640	3
41.63	1872	9	27.07	3488	15			
57.16	1776	9	28.19	3472	15			
63.94	1616	8	30.81	3408	15			
67.67	976	5	32.82	3392	15			
77.65	960	5	33.54	2912	13			
85.88	896	5	34.25	2848	13			
837.54	640	3	38.23	2832	13			
2137.82	400	2	38.72	1216	6			
			58.43	1136	6			
			59.73	992	5			
			61.97	960	5			
			77.76	880	5			
			132.52	800	4			
			133.69	720	4			
			242.50	640	4			
			577.38	560	3			

Table 6: *Instance 3 results comparing average excess workload and staffing cost from solving SINSA with three different approaches.*

SINSA-BD			SINSA-LRBD			SINSA-NBD		
Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses
8.43	1952	8	8.82	3664	15	2.55	2096	9
9.00	1856	8	8.93	3648	15	128.76	640	3
12.53	1792	8	9.26	3584	15			
13.53	1712	8	9.45	3072	13			
16.36	992	5	9.89	1552	7			
16.88	896	4	14.02	1056	5			
20.77	816	4	19.04	816	4			
31.67	672	3	28.46	560	3			
162.13	640	3	56.79	480	3			
			159.01	400	2			

Table 7: *Instance 4 results comparing average excess workload and staffing cost from solving SINSA with three different approaches.*

	SINSA-BD		SINSA-LRBD		SINSA-NBD	
Instance	Total	Nondominated	Total	Nondominated	Total	Nondominated
1	83	10	79	16	99	3
2	200	7	77	6	66	2
3	118	11	112	19	112	3
4	209	9	96	10	57	2

Table 8: *Total number of solutions and number of nondominated solutions.*

Policy 1			Policy 2		
Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses
32.35	2912	13	29.50	2912	12
35.37	2784	12	42.23	2512	11
56.00	1936	9	66.60	1952	9
59.64	1872	9	67.01	1872	9
68.07	1856	9	93.73	992	5
261.57	976	5	99.32	896	5
284.13	960	5	444.13	800	3
337.62	880	4	798.49	720	4
1132.64	720	3			
1680.28	576	2			

Table 9: *Instance 1 comparison of Policy 1 vs. Policy 2.*

work in the primary units resulting in better solutions. Thus, the algorithm quickly finds better solutions. In addition, regular nurses were knowledgable about patients' dignoses and more familiar with facilities and other procedures in their primary work units causing smaller excess workload for nurses. These results are consistent with academic literature [66]; nurses spend more time working in non-originally assigned units, since they spend much more time performing unit routines, searching for medical supplies, and caring for patients with unfamiliar diagnoses. A unit orientation, including unit routine introduction, patient care documentation overview, and assistants' phone numbers can help float nurses to reduce nervous tension, time, and workload, as well as increase quality of patient care [54, 66]. Instances 1 and 4 revealed the opposite results, floating regular nurses became helpful. The nurse supervisor can use this model along with her judgment to evaluate a float assignment policy based upon nurses' workload and staffing cost. As hospital administrations follow the right policy, they would reduce workload for nurses, increase care for patients, and reduce hospital costs.

6 Conclusions and Future Research

We described short-term nurse staffing and nurse-to-patient assignment. We integrated these two problems within a stochastic programming model with an objective to minimize an expected excess workload on nurses taking patient care uncertainty into consideration subject to the hard budget constraint. We provided three SINSA decompositions and solution algorithms based on the L-shaped method, which are (1) Benders' decomposition, (2) Lagrangian relaxation with Benders' decomposition, and (3) nested Benders' decomposition. We demonstrated that our model can be considered as a two-stage stochastic program for the first two approaches and a three-stage stochastic program for the last approach. As we intend to search for nondominated bicriteria solutions with the tradeoff between excess workload and staffing cost, we varied the right-hand side of the hard budget constraint in the Benders' decomposition and the

Policy 1			Policy 2		
Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses
14.14	1936	9	7.44	1776	9
24.26	1872	9	11.68	1376	7
33.39	1760	9	12.04	816	4
43.87	976	5	72.28	800	4
60.48	880	5			
707.45	800	3			
750.17	416	2			

Table 10: *Instance 2 comparison of Policy 1 vs. Policy 2.*

Policy 1			Policy 2		
Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses
21.71	2944	13	22.51	2848	13
24.60	2848	12	23.67	1936	9
30.53	1936	9	40.66	1888	9
41.63	1872	9	60.36	1872	9
57.16	1776	9	63.25	992	5
63.94	1616	8	68.35	976	5
67.67	976	5	76.50	896	5
77.65	960	5	165.09	880	4
85.88	896	5	851.04	816	3
837.54	640	3	1506.12	720	3
2137.82	400	2			

Table 11: *Instance 3 comparison of Policy 1 vs. Policy 2.*

Policy 1			Policy 2		
Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses
8.43	1952	8	3.95	1936	8
9.00	1856	8	6.55	1856	8
12.53	1792	8	6.77	1584	7
13.53	1712	8	10.58	976	5
16.36	992	5	12.54	960	5
16.88	896	4	17.43	880	4
20.77	816	4	22.77	832	4
31.67	672	3	47.94	800	3
162.13	640	3	134.16	416	2

Table 12: *Instance 4 comparison of Policy 1 vs. Policy 2.*

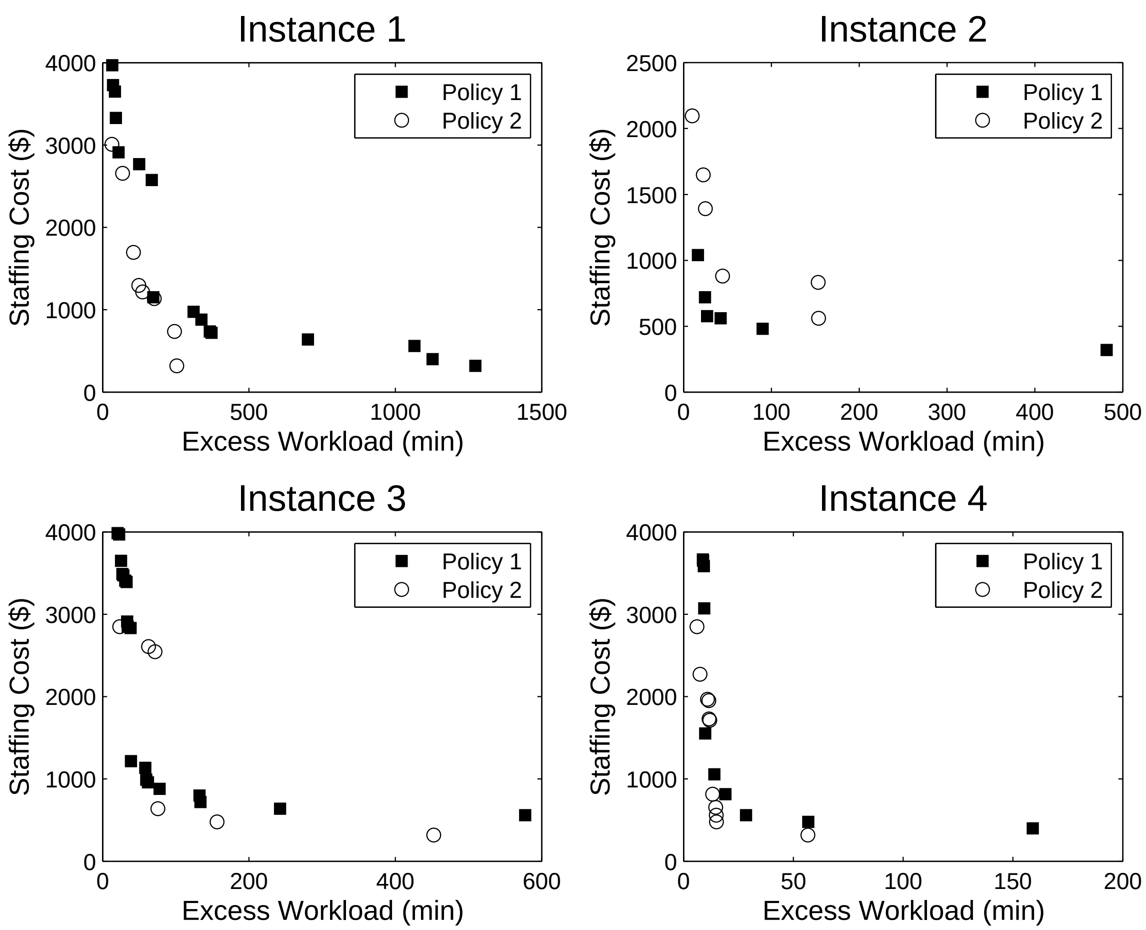

Figure 5: *Efficient frontiers comparing average excess workload and staffing cost of Policy 1 vs. Policy 2 for all four instances.*

nested Benders' decomposition approaches. In the Lagrangian relaxation with Benders' decomposition approach, we relaxed the budget constraint and penalized the staffing cost in the objective function. We use our algorithm approaches to find several nondominated nurse staffing and assignment solutions that minimize bicriteria objectives. We tested our model with four instances generated from real data from two medical-surgical units from the Northeast Texas hospital. Since the subproblems of these three solution approaches are similar to those from Punnakitikashem et al. [63], we employed their greedy algorithm to solve the subproblems. Instead of solving the problem optimally, we presented alternative non-optimal ways to obtain good staffing and assignment solutions. We solved our model with three solution methodologies, and we collected the nondominated solutions within 30 minutes. Computational results showed that SINSA-BD provided promising results with a predetermined budget, while SINSA-LRBD generated more and better nondominated solutions than the other two approaches with a flexible budget. Simultaneously considering nurse staffing and assignment is more desirable than doing them sequentially. We also provided efficient frontiers between excess workload and staffing cost of three solution approaches, which allow decision makers to select the right staffing policy. Moreover, we demonstrated that our model can be used to evaluate a float assignment policy based upon patients, available nurses of each type, and the budget for a shift.

Incorporating a nurse assignment within staffing decisions would likely provide better care for patients as well as balance workload for nurses. Hospitals also benefit from having better budget control, providing quality care to patients, and reducing liability cost. An integrated nurse staffing and assignment decision-support system that used our model would reduce the burden of the nursing shortage.

There are several interesting possibilities for future research. Because of the dynamic nature of the shift, patients are often admitted and discharged during a shift. One interesting topic is to consider how to assign newly admitted patients to nurses. Furthermore, our data included only primary diagnoses of patients, nurse types, and individual nursing skills. An extension to consider the following factors will likely to provide more accurate results.

- Multiple diagnoses. It is common for a patient to have multiple diagnoses during his/her stay in a hospital unit.
- Dynamic acuity. As the progress of a patient's condition changes over time, the acuity level is changed. Patients with different levels of acuity require different amounts of required care from nurses.
- Education levels of nurses. Education levels of nurses have an inverse relationship with mortality rates as well as adverse patient outcomes [31, 52].

Finally, incorporating the mid-term nurse scheduling into our model allowing feedback of a current shift for future corrections is another challenging area of research.

7 Acknowledgment

This research is supported by Robert Wood Johnson Foundation under Grant Number 053963. We would like to thank Terry Clark from Baylor Medical Center at Grapevine TX, for providing us data for this research.

References

[1] W. J. Abernathy, N. B., J. C. Hershey, and S. Wandel. A three-stage manpower planning and scheduling model–a service-sector example. *Operations Research*, 21(3):693–711, 1973.

[2] U. Aickelin and K. A. Dowsland. Exploiting problem structure in a genetic algorithm approach to a nurse rostering problem. *Journal of Scheduling*, 3(3):139–153, 2000.

[3] U. Aickelin and K. A. Dowsland. An indirect genetic algorithm for a nurse scheduling problem. *Computers & Operations Research*, 31(5):761–778, 2003.

[4] L. H. Aiken, S. P. Clark, D. M. Sloane, J. Sochalski, and J. H. Silber. Hospital nurse staffing and patient mortality, nurse burnout, and job dissatisfaction. *Journal of the American Medical Association*, 288(16):1987–1993, 2002.

[5] M. Anzai and Y. Miura. Computer program for quick work scheduling of nursing staff. *Medical Informatics*, 12:43–52, 1987.

[6] J. Arthur and A. Ravindran. A multiple objective nurse scheduling model. *IIE Transactions*, 13(1):55–60, 1981.

[7] M. N. Azaiez and S. S. A. Sharif. A 0-1 goal programming model for nurse scheduling. *Computers & Operations Research*, 32(3):491–507, 2005.

[8] J. F. Bard and H. W. Purnomo. Real-time scheduling for nurses in response to demand fluctuations and personnel shortages. In M.A. Trick and E.K. Burke, editors, *Proceedings of the 5th International Conference on the Practice and Theory of Automated Timetabling*, pages 67–87, Pittsburgh, PA, 2004.

[9] J. F. Bard and H. W. Purnomo. Hospital-wide reactive scheduling of nurses with preference considerations. *IIE Transactions*, 35:589–608, 2005.

[10] J. F. Bard and H. W. Purnomo. Short-term nurse scheduling in response to daily fluctuations in supply and demand. *Health Care Management Science*, 8:315–324, 2005.

[11] J. F. Benders. Partitioning procedures for solving mixed variables programming problems. *Numerische Mathematics*, 4:238–252, 1962.

[12] H. S. Berliner and E. Ginzberg. Why this hospital nursing shortage is different. *The Journal of the American Medical Association*, 288(21):27422744, 2002.

[13] I. Berrada, J. A. Ferland, and P. Michelon. A multi-objective approach to nurse scheduling with both hard and soft constraints. *Socio-Economic Planning Science*, 30(3):183–193, 1996.

[14] D. P. Bertsekas. *Nonlinear Programming: 2nd Edition*. Athena Scientific, 1999.

[15] J. R. Birge. Decomposition and partitioning methods for multistage stochastic linear programs. *Operations Research*, 33(5):989–1007, 1985.

[16] J. R. Birge and F. Louveaux. *Introduction to Stochastic Programming*. Springer, New York, New York, 1997.

[17] R. Blau and A. Sear. Nurse scheduling with a microcomputer. *Journal of Ambulance Care Management*, pages 1–13, 1983.

[18] M. J. Brusco and L. W. Jacobs. Cost analysis of alternative formulations for personnel scheduling in continuously operating organisations. *European Journal of Operational Research*, 86:249–261, 1995.

[19] E. K. Burke, P. D. Causmaecker, and G. V. Berghe. A hybrid tabu search algorithm for the nurse rostering problem. In *SEAL'98: Selected papers from the Second Asia-Pacific Conference on Simulated Evolution and Learning on Simulated Evolution and Learning*, pages 187–194, London, UK, 1999. Springer-Verlag.

[20] E. K. Burke, P. D. Causmaecker, G. V. Berghe, and H. V. Landeghem. The state of the art of nurse rostering. *Journal of Scheduling*, 7(6):441–499, November 2004.

[21] E. K. Burke, P. D. Causmaecker, S. Petrovic, and G. V. Berghe. A disaggregation model of a flexible nurse scheduling support system. In *Proceedings of Congress on Evolutionary Computation*, number CEC2002, pages 1197– 1202, Honolulu, HI, 2002. IEEE Press.

[22] E. K. Burke, P. Cowling, P. D. Caumaecker, and G. V. Berghe. A memetic approach to the nurse rostering problem. *Applied Intelligence special issue on Simulated Evolution and Learning*, 15(3):199–214, 2001.

[23] California Department of Health Services. State health director announces proposed changes to nurse-to-patient ratio regulations. http://www.applications.dhs.ca.gov/pressreleases/store/PressReleases/04-70.html, November 2004.

[24] B. Cheang, H. Li, A. Lim, and B. Rodrigues. Nurse rostering problems-a bibliographic survey. *European Journal of Operational Research*, 151(3):447–460, December 2003.

[25] J. G. Chen and T. Yeung. Hybrid expert system approach to nurse scheduling. *Computers in Nursing*, pages 183–192, 1993.

[26] S. P. Clarke. The policy implications of staffing-outcomes research. *Journal of Nursing Administration*, 35(1):17–19, 2004.

[27] J. F. Moore D. Cline, C. Reilly. Whats behind rn turnover? *Nursing Management*, 34(10):50–53, 2003.

[28] F. de Véricourt and O. B. Jennings. Nurse-to-patient ratios in hospital staffing: a queuing perspective. `http://faculty.fuqua.duke.edu/%7Efdv1/bio/ratios3.pdf`, 2006.

[29] K. Dowsland. Nurse scheduling with tabu search and strategic oscillation. *European Journal of Operational Research*, 106:393–407, 1998.

[30] K. A. Dowsland and J. M. Thompson. Solving a nurse scheduling problem with knapsacks, networks and tabu search. *Journal of the Operational Research Society*, 51:825–833, 2000.

[31] C. A. Estabrooks, W. K. Midodzi, G. C. Cummings, K. L. Ricker, and P. Giovannetti. The impact of hospital nursing characteristics on 30-day mortality. *Nursing Research*, 54(2):74–84, 2005.

[32] J. Baileyand J. Field. Personnel scheduling with flexshift models. *Journal of Operations Management*, 5:327–338, 1985.

[33] M. L. Fisher. The lagrangian relaxation method for solving integer programming problems. *Management Science*, 50(12):1861–1871, 2004.

[34] L. S. Franz, H. M. Baker, G. K. Leong, and T. R. Rakes. Mathematical model for scheduling and staffing multiclinic health regions. *European Journal of Operational Research*, 41(3):277–289, 1989.

[35] M. Held, P. Wolfe, and H.D. Crowder and. Validation of subgradient optimization. *Mathematical Programming*, 6:62–88, 1974.

[36] IOM. *Keeping patients safe: Transforming the work environment of nurses*. National Academy Press, Washington, DC, 2004.

[37] M. Isken. An implicit tour scheduling model with applications in healthcare. *Annals of Operations Research*, 128:91–109, 2004.

[38] M. W. Isken and M. W. Hancock. Tactical scheduling analysis for hospital ancillary units. *Journal of the Society for Health Systems*, 5(4):11–24, 1998.

[39] A. Jaszkiewicz. A metaheuristic approach to multiple objective nurse scheduling. *Foundations of Computing and Decision Sciences*, 22(3):169–184, 1997.

[40] B. Jaumard, F. Semet, and T.Vovor. A generalized linear programming model for nurse scheduling. *European Journal of Operational Research*, 107(1):1–18, 1998.

[41] H. Kawanaka, K. Yamamoto, T. Yoshikawa, T. Shinogi, and S. Tsuruoka. Genetic algorithm with the constraints for nurse scheduling problem. In *Proceedings of Congress on Evolutionary Computation*, pages 1123–1130, Seoul, 2001. IEEE Press.

[42] G. Kazahaya. Harnessing technology to redesign labor cost management reports. *Healthcare Financial Management*, 59(4):94–100, 2005.

[43] M. M. Kostreva and K. S. B. Jennings. Nurse scheduling on a microcomputer. *Computers & Operations Research*, 18(9):731–739, December 1991.

[44] A. J. Lankshear, T. A. Sheldon, and A. Maynard. Nurse staffing and healthcare outcomes a systematic review of the international research evidence. *Advances in Nursing Science*, 28(2):163–174, 2005.

[45] H. Li, A. Lim, and B. Rodrigues. A hybrid ai approach for nurse rostering problem. In *Proceedings of the 2003 ACM symposium on Applied computing*, Melbourne, FL, 2003. IEEE Press.

[46] H. H. Millar and M. Kiragu. Cyclic and non-cylcic scheduling of 12h shift nurses by network programming. *European Journal of Operational Research*, 104(3):582–592, 1998.

[47] H. E. Miller, W. P. Pierskalla, and G. J. Rath. Nurse scheduling using mathematical programming. *Operations Research*, 24(5):857–870, 1976.

[48] M. Moz and M. Pato. Solving the problem of rerostering nurse schedules with hard constraints: New multicommodity flow models. *Annals of Operations Research*, 128:179–197, 2004.

[49] M. Moz and M. Pato. A genetic algorithm approach to a nurse rerostering problem. *Computers & Operations Research*, 34:667–691, 2007.

[50] C. Mullinax and M. Lawley. Assigning patients to nurses in neonatal intensive care. *Journal of the Operational Research Society*, 53(1):25–35, 2002.

[51] A. Musa and U. Saxena. Scheduling nurses using goal-programming techniques. *IIE Transactions*, 16(3):216–221, 1984.

[52] J. Needleman, P. I. Buerhaus, M. Stewart S. Mattke, and K. Zelevinsky. Nurse-staffing levels and the quality of care in hospitals. *New England Journal of Medicine*, 346(22):1715–1722, 2002.

[53] G. L. Nemhauser and L. A. Wolsey. *Integer and Combinatorial Optimization*. John Wiley & Sons, Inc., 1988.

[54] D. Nicholls, E. Duplaga, and L. Meyer. Nurses' attities about floating. *Nursing Management*, 27(1):56–58, 1996.

[55] M. Okada. An approach to the generalized nurse scheduling problem: generation of a declarative program to represent institution-specific knowledge. *Computers and Biomedical Research*, 25(5):417–434, 1992.

[56] M. Okada and M. Okada. Prolog-based system for nursing staff scheduling implemented on a personal computer. *Computers and Biomedical Research*, 21(1):53–63, 1988.

[57] R. O'Neill. Nested decomposition for multistage convex programs. *SIAM Journal of Control and Optimization*, 14:409–418, 1976.

[58] F. Overfelt. Nursing ratios–are they good or bad? In *Proceedings of The 16th Annual Society for Health Systems Management Engineering Forum, Orlando, Florida*, 2004.

[59] I. Ozkarahan. A disaggregation model of a flexible nurse scheduling support system. *Socio-Economical Planning Science*, 25(1):9–26, 1991.

[60] I. Ozkarahan and J. E. Bailey. Goal programming model subsystem of a flexible nurse scheduling support system. *IIE Transactions*, 16:306–316, 1988.

[61] J. Powers. Accepting and refusing assignments. *Nursing Management*, 24(9):64–73, September 1993.

[62] P. Punnakitikashem. *Integrated Nurse Staffing and Assignment under Uncertainty*. PhD thesis, The University of Texas at Arlington, Department of Industrial and Manufacturing Systems Engineering, 2007.

[63] P. Punnakitikashem, J. M. Rosenberger, and D. B. Behan. Stochastic programming for nurse assignment. To appear in *Computational Optimization and Applications*, 2006.

[64] J. M. Rosenberger, D. B. Green, B. Keeling, and P. G. Turpin. Optimizing nurse assignment. In *Proceedings of The 16th Annual Society for Health Systems Management Engineering Forum*, Orlando, FL, 2004.

[65] E. S. Rosenbloom and N. F. Goertzen. Nurse scheduling using mathematical programming. *Operations Research*, 31:857–870, 1976.

[66] S. Rudy and J. Sions. Floating managing a recruitment and retention issue. *Journal of Nursing Administration*, 33(4):196–198, 2003.

[67] S. Scott and R. Simpson. Case-bases incorporating scheduling constraint dimensions - experiences in nurse rostering. In *Proceedings of the 4th European Workshop on Advances in Case-Based Reasoning*, pages 392–401, September 1998.

[68] S. P. Siferd and W. C. Benton. A decision model for shift scheduling of nurses. *European Journal of Operational Research*, 74(3):519–527, 1994.

[69] D. Sitompul and S. Randhawa. Nurse scheduling models: a state-of-the-art review. *Journal of the Society for Health Systems*, 2(1):62–72, 1990.

[70] P. L. Stamps. *Nurses and work satisfaction: An index for measurement, second edition*. Chicago: Health Administration Press, 1997.

[71] M. W. Stanton. Hospital nurse staffing and quality of care. Technical Report 04-0029, Agency for Healthcare Research and Qualitys. Research in Action, AHRQ, March 2004.

[72] V. M. Trivedi and M. Warner. A branch and bound algorithm for optimum allocation of float nurses. *Management Science*, 22(9):972–981, 1976.

[73] L. Walts and A. S. Kapadia. Patient classification system: An optimization approach. *Health Care Management Review*, 21:75–82, 1996.

[74] D. M. Warner. Scheduling nursing personnel according to nursing preferences: A mathematical programming approach. *Operations Research*, 24(5):842–856, 1976.

[75] D. M. Warner and J. Prawda. A mathematical programming model for scheduling nursing personnel in a hospital. *Management Science*, 19(4):411–422, 1972.

[76] M. Warner, B. Kelle, and S. Martel. Automated nurse scheduling. *Journal of the Society for Health Systems*, 2(2):66–80, 1990.

[77] P. D. Wright, K. M. Bretthauer, and M. J. Côté. Short-term workforce management: Cross-training, scheduling and allocation of heterogeneous nurses in a hospital. In *Proceedings of Annual Meeting of the Decision Sciences Institute*, pages 1672–1677, San Diego, CA, USA, November 2002. Decision Sciences Institute.

[78] P. D. Wright, K. M. Bretthauer, and M. J. Côté. Reexamining the nurse scheduling problem: Staffing ratios and nursing shortages. *Decision Sciences*, 37(1):39–70, 2006.

30 sec.			60 sec.			120 sec.			300 sec.		
Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses
58.90	3152	14	49.17	3168	14	37.05	3904	17	32.83	3968	17
102.11	3088	14	67.66	2032	9	38.94	3728	16	35.48	3728	16
185.30	2768	13	93.05	1440	7	39.99	3648	16	42.12	3648	16
309.24	928	4	104.78	1392	7	42.41	3232	14	45.09	3328	15
311.24	800	4	117.56	1360	6	75.96	3088	14	54.60	2912	14
545.67	720	4	126.23	1200	6	117.67	1200	6	125.21	2768	14
659.46	656	3	195.67	1120	6	133.87	1120	6	168.12	2576	12
688.15	560	3	221.31	1072	5	218.26	1056	6	172.93	1152	6
890.41	480	3	308.52	912	4	255.22	976	5	311.06	976	5
1094.86	400	2	336.74	816	4	326.56	896	4	337.68	880	4
1273.56	320	2	375.79	720	4	359.56	720	4	366.12	736	4
			761.68	560	3	1048.90	560	3	372.15	720	4
			1127.59	400	2	1094.86	400	2	701.54	640	4
			1273.56	320	2	1273.56	320	2	1065.57	560	3
									1127.59	400	2
									1273.56	320	2

Table 13: *Instance 1: solving SINSA-BD with different time limits within SINSA-LRBD.*

[79] N. Yankovic and L. V. Green. A queueing model for nurse staffing. Technical Report DRO-2007-09, Columbia University, New York, NY, August 2007.

8 Appendix

In the appendix, we state additional tables from parameter tuning in Section 5.2.1 and from solving SINSA with three different approaches in Section 5.2.

30 sec.			60 sec.			120 sec.			300 sec.		
Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses
234.34	2400	12	18.40	480	3	34.12	3536	16	16.51	1040	5
607.06	2240	11	105.81	320	2	44.77	3216	15	24.35	720	4
730.01	1520	9				50.16	2976	14	26.92	576	3
						59.06	2656	13	42.25	560	3
						71.59	2640	13	90.17	480	3
						91.40	2480	12	481.57	320	2
						107.13	320	2			

Table 14: *Instance 2: solving SINSA-BD with different time limits within SINSA-LRBD.*

30 sec.			60 sec.			120 sec.			300 sec.		
Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses
39.79	3152	14	28.46	3168	14	33.54	2912	13	20.42	3984	17
73.98	896	5	44.80	2272	10	54.22	2608	12	22.57	3968	17
94.04	880	5	56.13	1952	9	57.53	2592	12	25.22	3648	16
152.28	720	4	60.97	1472	7	85.76	2336	11	27.07	3488	15
259.58	576	3	76.47	1456	7	286.35	2096	10	28.19	3472	15
310.57	560	3	79.55	1280	7	366.15	1696	9	30.81	3408	15
335.70	480	3	86.05	1200	6	941.84	416	2	32.82	3392	15
818.26	416	2	117.92	1136	6				33.54	2912	13
941.84	400	2	146.25	720	4				34.25	2848	13
			251.79	560	3				38.23	2832	13
			391.33	480	3				38.72	1216	6
			818.26	400	2				58.43	1136	6
			1886.86	320	2				59.73	992	5
									61.97	960	5
									77.76	880	5
									132.52	800	4
									133.69	720	4
									242.50	640	4
									577.38	560	3

Table 15: *Instance 3: solving SINSA-BD with different time limits within SINSA-LRBD.*

30 sec.			60 sec.			120 sec.			300 sec.		
Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses
19.82	2128	10	17.39	2096	9	11.63	3072	13	8.82	3664	15
32.77	1136	6	19.80	1936	9	13.12	1712	8	8.93	3648	15
74.81	960	5	32.77	1136	6	21.34	1632	8	9.26	3584	15
134.09	896	5	74.81	960	5	32.77	1136	6	9.45	3072	13
159.01	880	5	134.09	896	5	74.81	960	5	9.89	1552	7
			159.01	880	5	134.09	896	5	14.02	1056	5
						159.01	880	5	19.04	816	4
									28.46	560	3
									56.79	480	3
									159.01	400	2

Table 16: *Instance 4: solving SINSA-BD with different time limits within SINSA-LRBD.*

30 sec.			60 sec.			120 sec.			300 sec.		
Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses
428.37	2992	13	149.33	2992	13	75.98	2992	13	37.89	2992	13
825.08	480	3	785.97	720	4	758.97	720	4	620.64	800	4
			817.76	480	3	815.15	480	3	675.24	560	3

Table 17: *Instance 1: solving the retricted master problem (RMP^1) with different time limits within SINSA-NBD.*

30 sec.			60 sec.			120 sec.			300 sec.		
Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses
56.24	2832	13	12.86	2832	13	12.86	2832	13	12.86	2832	13
58.54	1200	6	38.57	1376	6	21.27	1280	6	44.22	1456	7
1203.30	880	4	1195.89	896	4	1195.45	800	4			

Table 18: *Instance 2: solving the retricted master problem (RMP^1) with different time limits within SINSA-NBD.*

30 sec.			60 sec.			120 sec.			300 sec.		
Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses
137.90	2864	11	53.13	2864	11	50.29	2864	11	24.81	2864	11
171.37	880	4	156.21	880	4	149.22	880	4	138.96	896	4
228.54	640	3	601.31	720	3	224.79	640	3	218.17	640	3

Table 19: *Instance 3: solving the retricted master problem (RMP^1) with different time limits within SINSA-NBD.*

30 sec.			60 sec.			120 sec.			300 sec.		
Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses	Excess workload (min)	Staffing cost ($)	No. of nurses
18.63	2096	9	9.16	2096	9	6.40	2096	9	2.55	2096	9
128.89	640	3	128.76	640	3	128.76	640	3	128.76	640	3

Table 20: *Instance 4: solving the retricted master problem (RMP^1) with different time limits within SINSA-NBD.*

Excess workload (min)	Staffing cost ($)	No. of nurses	Med-Surg1 No. of Reg	PRN	OT	Agen	Med-Surg2 No. of Reg	PRN	OT	Agen
SINSA-BD										
32.68	2912	13	3	0	1	0	2	2	3	2
35.37	2784	12	3	0	1	0	1	4	1	2
56.00	1936	9	2	0	0	0	2	1	3	1
59.64	1872	9	3	0	0	0	1	2	3	0
68.07	1856	9	2	0	1	0	2	1	3	0
261.57	976	5	2	0	0	0	1	1	1	0
284.13	960	5	2	0	0	0	1	0	2	0
337.62	880	4	1	0	0	0	1	0	1	1
1132.64	720	3	1	0	0	1	0	0	1	0
1680.28	576	2	0	1	0	0	0	0	0	1
SINSA-LRBD										
32.83	3968	17	3	1	0	1	3	2	4	3
35.48	3728	16	3	1	1	1	3	2	2	3
42.12	3648	16	3	0	1	1	3	3	3	2
45.09	3328	15	3	1	2	1	3	2	2	1
54.60	2912	13	3	1	0	0	3	1	2	3
125.21	2768	13	3	0	1	0	3	3	2	1
168.12	2576	12	3	0	0	1	3	1	3	1
172.93	1152	6	2	0	0	0	2	2	0	0
311.06	976	5	2	0	0	0	1	1	1	0
337.68	880	4	1	0	0	0	1	0	1	1
366.12	736	4	1	0	0	0	2	1	0	0
372.14	720	4	1	0	0	0	2	0	1	0
701.54	640	4	1	0	0	0	3	0	0	0
1065.57	560	3	1	0	0	0	1	0	1	0
1127.59	400	2	1	0	0	0	0	0	1	0
1273.56	320	2	1	0	0	0	1	0	0	0
SINSA-NBD										
37.89	2992	13	2	1	1	1	3	1	2	2
620.64	800	4	1	0	1	0	1	0	1	0
675.24	560	3	1	0	0	0	1	0	1	0

Table 21: *Instance 1 results from solving SINSA with three different approaches.*

Excess workload (min)	Staffing cost ($)	No. of nurses	Med-Surg1 No. of Reg	PRN	OT	Agen	Med-Surg2 No. of Reg	PRN	OT	Agen
SINSA-BD										
14.14	1936	9	3	0	0	0	2	1	1	2
24.26	1872	9	2	1	0	0	3	1	1	1
33.39	1760	9	3	0	0	0	3	0	2	1
43.87	976	5	1	0	0	0	2	1	1	0
60.48	880	5	1	0	0	0	3	0	1	0
707.45	800	3	1	0	0	0	0	0	0	2
750.17	416	2	1	0	0	0	0	1	0	0
SINSA-LRBD										
16.51	1040	5	1	0	1	0	2	0	0	1
24.35	720	4	1	0	0	0	2	0	1	0
26.92	576	3	1	0	0	0	1	1	0	0
42.25	560	3	1	0	0	0	1	0	1	0
90.17	480	3	1	0	0	0	2	0	0	0
481.57	320	2	1	0	0	0	1	0	0	0
SINSA-NBD										
12.86	2832	13	2	0	0	0	4	2	3	2
44.22	1456	7	2	1	0	1	2	0	1	0

Table 22: *Instance 2 results from solving SINSA with three different approaches.*

Excess workload (min)	Staffing cost ($)	No. of nurses	Med-Surg1 No. of Reg	PRN	OT	Agen	Med-Surg2 No. of Reg	PRN	OT	Agen
SINSA-BD										
21.71	2944	13	3	1	0	1	1	3	4	0
24.60	2848	12	3	0	1	0	1	3	1	3
30.53	1936	9	3	0	0	0	2	1	1	2
41.63	1872	9	3	0	1	0	1	2	2	0
57.16	1776	9	3	0	0	0	3	1	1	1
63.94	1616	8	2	0	0	0	2	1	3	0
67.67	976	5	2	0	0	0	1	1	1	0
77.65	960	5	2	0	0	0	2	0	0	1
85.88	896	5	2	0	0	0	2	1	0	0
837.54	640	3	1	0	1	0	0	0	1	0
2137.82	400	2	0	0	1	0	1	0	0	0
SINSA-LRBD										
20.42	3984	17	3	1	1	0	3	3	2	4
22.57	3968	17	3	1	1	1	3	2	3	3
25.22	3648	16	3	1	0	2	3	2	4	1
27.07	3488	15	3	0	1	3	3	3	1	1
28.19	3472	15	3	0	1	0	3	2	2	4
30.81	3408	15	3	0	1	0	3	3	2	3
32.82	3392	15	3	2	1	1	3	0	3	2
33.54	2912	13	3	1	0	0	3	1	2	3
34.25	2848	13	3	1	1	0	3	2	1	2
38.23	2832	13	3	0	1	0	3	2	2	2
38.72	1216	6	2	0	0	0	1	1	2	0
58.43	1136	6	2	0	0	0	2	1	1	0
59.73	992	5	2	0	0	0	1	2	0	0
61.97	960	5	2	0	0	0	1	0	2	0
77.76	880	5	2	0	0	0	2	0	1	0
132.52	800	4	2	0	0	0	1	0	0	1
133.69	720	4	2	0	0	0	1	0	1	0
242.50	640	4	2	0	0	0	2	0	0	0
577.38	560	3	1	0	0	0	1	0	1	0
SINSA-NBD										
24.81	2864	11	1	3	1	0	1	1	0	4
138.96	896	4	1	1	0	0	1	0	0	4
218.17	640	3	1	0	0	0	1	0	0	1

Table 23: *Instance 3 results from solving SINSA with three different approaches.*

Excess workload (min)	Staffing cost ($)	No. of nurses	Med-Surg1 No. of Reg	Med-Surg1 No. of PRN	Med-Surg1 No. of OT	Med-Surg1 No. of Agen	Med-Surg2 No. of Reg	Med-Surg2 No. of PRN	Med-Surg2 No. of OT	Med-Surg2 No. of Agen
SINSA-BD										
8.43	1952	8	1	0	1	1	1	2	1	1
9.00	1856	8	1	0	1	0	2	1	1	2
12.53	1792	8	2	0	0	0	1	2	2	1
13.53	1712	8	1	0	2	0	2	2	1	0
16.36	992	5	1	0	0	0	2	2	0	0
16.88	896	4	1	0	0	0	0	1	2	0
20.77	816	4	1	0	0	0	1	1	1	0
31.67	672	3	1	0	0	0	0	2	0	0
162.13	640	3	1	0	1	0	0	0	1	0
SINSA-LRBD										
8.82	3664	15	2	2	1	1	2	2	2	3
8.93	3648	15	2	0	0	0	2	3	4	4
9.26	3584	15	2	1	2	0	2	3	2	3
9.45	3072	13	2	1	1	2	2	1	3	1
9.89	1552	7	2	0	0	0	1	2	1	1
14.02	1056	5	1	0	0	0	1	1	2	0
19.04	816	4	1	0	0	0	1	1	1	0
28.46	560	3	1	0	0	0	1	0	1	0
56.79	480	3	1	0	0	0	2	0	0	0
159.01	400	2	1	0	0	0	0	0	1	0
SINSA-NBD										
2.55	2096	9	2	0	0	1	2	1	1	2
128.76	640	3	1	0	0	1	1	0	0	0

Table 24: *Instance 4 results from solving SINSA with three different approaches.*