

# Optimizing Selection of Technologies in a Multiple Stage, Multiple Objective Wastewater Treatment System

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## Abstract

In this paper, a multiple stage wastewater treatment system (WTS) is solved for the selection of technological options at each stage to *minimize* (economic cost, size, odor emissions) and to *maximize* (nutrient recovery, robustness, global desirability). Stages in the wastewater treatment system are the levels of treatment. There are seventeen levels of treatment, where the first eleven levels are for the liquid treatment and the last six levels are for the solid treatment. This results in a 20-dimensional, continuous-state, 17-stage, 6-objective, stochastic optimization problem. The resulting multiple stage, multiple objective (MSMO) WTS is solved using the three-phase methodology in conjunction with the multiple objective version of high-dimensional, continuous-state, stochastic dynamic programming (SDP). The three-phase methodology comprises *the input phase*, *the matrix generation phase*, and *the weighting phase*. The primary goal of three-phase methodology is to obtain weight vectors at each stage of the WTS utilizing expert's opinions in the input phase, computing pairwise comparison matrices at each stage using the geometric-mean based methods in the matrix generation phase, and then calculating weight vectors at each stage using the eigenvector method in the weighting phase. The weight vectors are then used to scalarize the vector optimization problem, which is solved using the high-dimensional, continuous-state SDP augmented for handling multiple objectives at each stage.

The results obtained are practical as evidenced by the selection of new technologies in levels 1 and 5 thereby validating expert's decision to include them in the evaluation process. In addition to encouraging reviews from WTS experts, the implementation results satisfy a set of external constraints in the form of interstage dependencies between technological options in the WTS. Furthermore, the solution technique presented here utilizes *expert's opinions* in the solution development process, and is quite *general* in its application to a variety of large-scale MSMO problems.

**Key Words:** Multiple objective decisions, Stochastic dynamic programming, Wastewater treatment model, Pareto scalarization, Analytic hierarchy process

# 1 Introduction

Rapid population growth and continued industrial development have created enormous challenges in conserving water and making it potable. However, any wastewater treatment system (WTS) needs to be designed to meet the economic, environmental, space, and performance requirements. The wastewater treatment system considered here is based on the work of Chen and Beck (1997), which involves removing the liquid and solid pollutants from domestic wastewater in several levels of processing. This WTS comprises eleven levels of liquid processing and six levels of solid processing.

To meet the current domestic water requirements the liquid processing line of the WTS has been modified from the one used by Chen and Beck as follows:

- Yellow Water Separation, and Yellow and Black Water Separation are two new wastewater treatment technologies that have been added in level 1 of the WTS.
- Upflow Anaerobic Sludge Blanket (UASB) System *plus* Activated Sludge Process (C, P, N) is a new treatment technology added in level 5 of the WTS.

The liquid and solid processing lines of the WTS can be seen in Fig. 1 and Fig. 2, respectively.

Tsai, Chen, Beck and Chen (2004) presented a decision-making framework to evaluate current and emerging technologies for the multi-level wastewater treatment system with the objective to minimize economic cost (capital cost *plus* operating cost). They solved a high-dimensional, continuous-state, multistage (where stages refer to levels of wastewater treatment) decision-making problem. Methodology and results are detailed in Tsai (2002). Unfortunately, the results were far from practical as it only considered the economics of the WTS and ignored a range of critical factors such as environment, size, performance, etc. A real-world wastewater treatment system must be designed to meet various objectives simultaneously, including the conflicting ones. Hence, the single objective WTS needs to be extended to a multiple objective version.

This paper improves earlier WTS versions by considering multiple objectives at each level of WTS. The following six objectives are considered at each WTS level: *minimize* (economic cost, size, odor emissions), and *maximize* (nutrient recovery, robustness, global desirability). This results in a high-dimensional, continuous-state, 17-stage, 6-objective optimization problem. Each level in the liquid processing line has a 20-dimensional state vector, and each level in the solid processing line has a 10-dimensional state vector. The resulting problem is apparently the largest multiple stage, multiple objective (MSMO) optimization problem in the current literature. To solve the resulting MSMO optimization problem, we present an approach that effectively combines three-phase methodology discussed in Tarun et al. (2007, 2008) and Tarun (2008) with the high-dimensional, continuous-state stochastic dynamic programming method augmented to handle the multiple objectives at each stage (or level) of WTS. As a result, the solution is practical and the approach has a high likelihood of success in making decisions in a real-world wastewater treatment system.

The objective of this paper is to evaluate wastewater treatment technologies, including new and emerging ones, in a multiple stage (or multi-level) wastewater treatment system based on a set of diverse criteria. Further, the presence of continuous state variables, high-dimensional state vectors, and uncertainty due to new and emerging wastewater treatment technologies compounds the complexity in solving an already complex multiple stage, multiple objective decision-making problem. Therefore, it is essential to involve decision makers in the solution process to obtain a solution that can be implemented in the real-world.

The three-phase methodology of Tarun (2008) allows decision makers to participate in the solution development process. It involves eliciting system knowledge from experts (the input phase),

utilizing experts' system knowledge to compute pairwise comparison matrices contrasting all possible pairs of objectives at each stage (the matrix generation phase), and obtaining weight vectors reflecting decision-makers' opinions for all the stages (the weighting phase). The results from the weighting phase are used to scalarize the MSMO wastewater treatment system. The purpose of scalarization is to convert the MSMO problem into a multiple stage, single objective problem without loss in any aspects of the original problem.

To solve the resulting multiple stage, single objective problem, the stochastic dynamic programming (SDP) approach in Tsai (2002) and Tsai et al. (2004) was augmented to handle multiple objectives at each stage. The modifications were made to capture expert opinions that lead to meaningful weight vectors in the weighting phase of our three-phase methodology. A routine was added to handle different units associated with multiple objectives. Few other changes were made to accommodate the newly added technology units mentioned above, and these were state transition equations, constraints, etc. The three-phase methodology in combination with orthogonal array-based experimental designs (for state space discretization) and multiple adaptive regression spline (MARS) technique (for the future value approximation in the stochastic dynamic programming model) led to practical results. Chen, Ruppert and Shoemaker (1999) showed that the combination of orthogonal array-based experimental designs and MARS provides a polynomial algorithm for numerically solving continuous-state SDP, and hence is appropriate for high-dimensional problems.

The practicality of results can be seen in the selection of new technologies in levels 1 and 5, validating their inclusion in the evaluation process. The results also satisfy interstage dependencies between various technology units across the wastewater treatment system. Furthermore, our wastewater treatment experts had encouraging reviews about the results. The detailed interpretation of results can be seen later. In summary, this paper presents a new approach to evaluate selection of technologies in a multistage multiobjective wastewater treatment system that

- addresses the disconnect between decision makers and solution developers,
- extends the analytic hierarchy process (AHP) to an MSMO decision-making domain,
- helps develop methodologies for the computation of pairwise comparison matrices that have traditionally been heavily dependent on direct inputs from experts,
- augments the high dimensional, continuous-state stochastic dynamic programming approach to handle multiple objectives at each stage, and
- solves a 20-dimensional, 17-stage, 6-objective, continuous-state optimization problem, which is larger than any numerically solved multistage multiobjective optimization problem in the literature.

The paper is organized as follows. Section 2 describes the decision-making elements in the multiple stage, multiple objective (MSMO) wastewater treatment system and presents the formulation for MSMO decision-making problem. Section 3 gives a detailed account of the solution methodologies. Section 4 presents the implementation results and discussions. Section 5 summarizes the contributions, discusses the practical validation, and gives an insight into the future research work needed to refine all three phases of our three-phase methodology.

## 2 Multiple Stage, Multiple Objective Wastewater Treatment System

In extending Tsai's model (Tsai, 2002) to an MSMO problem, the stages correspond to different levels of processing in WTS. There are seventeen levels of sequential treatment. The first eleven levels form the liquid processing line and the last six levels form the sludge processing line. The key decision-making elements in the MSMO formulation of WTS are presented next.

### 2.1 State variables

Given the stage of domestic wastewater treatment, the domestic wastewater has certain types of pollutants at certain levels. The levels of these pollutants go down when the wastewater is getting cleaned, or they go up when the wastewater is monitored, but not cleaned. The levels of pollutants in the wastewater at a given stage of wastewater treatment are referred to as states, and the pollutant types are termed as state variables. Formally, *state variables* represent the state of the wastewater treatment system going across various stages of treatment. At each stage of the liquid processing line, the following ten state variables were considered for the liquid pollutants:

1. chemical oxygen demand (**Liq-COD**)
2. suspended solids (**Liq-SS**)
3. organic-nitrogen (**Liq-orgN**)
4. ammonia-nitrogen (**Liq-ammN**)
5. nitrate-nitrogen (**Liq-nitN**)
6. total phosphorus (**Liq-totP**)
7. heavy metals (**Liq-HM**)
8. synthetic organic chemicals (**Liq-SOCs**)
9. pathogens (**Liq-pathogens**)
10. viruses (**Liq-viruses**)

Similarly, the following ten state variables were considered for the solid (or sludge) pollutants at each stage of both liquid and solid processing lines:

11. sludge volume (**Sl-Vol**)
12. sludge water content (**Sl-WC**)
13. sludge organic-carbon (**Sl-orgC**)
14. sludge inorganic-carbon (**Sl-inorgC**)
15. sludge organic-nitrogen (**Sl-orgN**)
16. sludge ammonia-nitrogen (**Sl-ammN**)
17. sludge total phosphorus (**Sl-totP**)

18. sludge heavy metals (**SI-HM**)
19. sludge synthetic organic chemicals (**SI-SOCs**)
20. sludge pathogens (**SI-pathogens**)

The state variables are *continuous*. All twenty state variables are monitored in the liquid processing line comprising levels 1 through 11. The liquid pollutants (or the ten liquid state variables with prefix ‘Liq’) are removed in levels 1 through 11 while the solid pollutants (or the ten sludge state variables with prefix ‘Sl’) are collected in levels 1, 2, 5, 6, and 7 of the liquid processing line. Further, the solid processing line from levels 12 through 17 removes the ten solid pollutants. The twenty-dimensional state vector at level  $\tau$ ,  $\mathbf{x}_\tau = (x_{\tau,1}, \dots, x_{\tau,20})'$ , where the first ten are liquid state variables and the last ten are sludge state variables.

## 2.2 Decision variables

At each stage of the wastewater treatment system, given the state of pollutants entering the stage, a decision needs to be made about the wastewater treatment technology that *minimizes* economic cost, size, and odor emissions, but *maximizes* nutrient recovery, robustness, and global desirability. In other words, there is a need to evaluate a collection of appropriate wastewater technologies at each stage of treatment to select a technology that best optimizes a set of diverse objectives simultaneously. Formally, *decision variables* are the technological options that are being evaluated at each stage of the multiple stage, multiple objective wastewater treatment system. To maintain consistency across various stages of processing the liquid treatment line and the solid treatment line are connected with stages numbered sequentially from 1 through 17. Of these seventeen treatment stages (or levels), stages 1 through 11 form the liquid treatment line and stages 12 through 17 form the solid processing line. The decision variables for the 17-stage WTS are shown in Table 1. At each stage (or level of processing), a decision has to be made regarding the selection of a technology unit. At any level, technology units can be added or removed depending on current or future needs. Also, an “empty unit” can be seen in all other WTS levels except the first level of the liquid line. The selection of “empty unit” at a particular level implies that no treatment is performed at that level. Further, the decision-making must account for the interstage dependencies that occur between certain levels of treatment, and thereby adding a few major constraints in the selection of technological options in these levels. Interstage dependencies between technology units at various levels of wastewater treatment are as follows.

1. In level 5, “Reed Bed System” requires using a technology unit in level 2.
2. In level 13, “Sludge Carver-Greenfield (C-G) Drying” requires using a technology unit in level 12.
3. In level 13, “Anaerobic Digestion” requires using a technology unit in level 12.
4. In level 14, “Filter and Belt” requires using the “Sludge Thickening Tank” in level 12 AND one of the following in level 13: Sludge Vertech + Ammonia Stripping, Catalytic Wet Oxidation Process (CWOP)-Upflow Anaerobic Sludge Blanket (UASB) + Ammonia Stripping, Sludge Hydrolysis + UASB, Anaerobic Digestion, OR Aerobic-Anaerobic Digestion.

Also, uncertainty of the following two types are modeled:

1. Uncertainty in concentration of the influent and values of the state variables at subsequent levels of WTS, and

2. Uncertainty in the performance of a technology, denoted by  $u_t$  in level  $t$ , including the *objectives* associated with a technology.

The uncertainty of type (1) is represented by range limits, initialized by the values in Table 2. All twenty state variables entering each level of the WTS are bounded by lower and upper limits. The lower limits at a level are computed based on the maximum possible pollutant removal, and the upper limits are based on the minimum possible pollutant removal assuming that the “empty unit” was not selected. To solve the resulting stochastic dynamic programming (SDP), a *statistical experimental design* based approach is used to efficiently represent the possible values of state variables and to help construct a model over the continuous ranges of the state variables. Type (2) uncertainty is represented by the stochastic vector  $\epsilon_{t,u_t}$ . The dimension of this vector depends on the performance parameters for a particular technology, specified in the database by Chen (1993). Since nothing is known about the appropriate probability distributions to represent the stochasticity, only the ranges of the performance parameters are specified, and sampling based on a uniform distribution is utilized. Narrower ranges are assigned to well-known technologies, while wider ranges are assigned to newer and emerging technologies.

### 2.3 Transition functions

The transition function for a particular level determines how the state variables change at the exit point of this level. For the multivariate transition function at level  $\tau$ , given  $\mathbf{x}_\tau$ , the state entering level  $\tau$ , if  $u_\tau$  is the treatment technology selected in level  $\tau$ , and  $\epsilon_{\tau,u_\tau}$  is the uncertainty associated with the transition, then the new state exiting level  $\tau$ ,  $\mathbf{x}_{\tau+1} = f_\tau(\mathbf{x}_\tau, u_\tau, \epsilon_{\tau,u_\tau})$ .

### 2.4 Objectives

As mentioned above, the six objective functions that are considered for the MSMO wastewater treatment system are the following.

1. Minimize economic cost (in *US Dollars*), capital and operating cost of the treatment technology units.
2. Minimize size (in  $m^2$ ), the land area occupied by the treatment technology units.
3. Minimize odor emissions (in  $mg/min$ ), obtained by multiplying the concentration of the discharged gas (in the unit of  $mg/l$  or  $mg/m^3$ ) and the flow of the gas (in the unit of  $l/min$  or  $m^3/min$ ).
4. Maximize nutrient recovery (on 1-5 scale), characterizing the rating of the treatment technology units in removing liquid or sludge pollutants.
5. Maximize robustness (no units), characterizing the insensitivity to the variation of the inputs.
6. Maximize global desirability (on 1-6 scale), characterizing the impact of wastewater treatment outputs on the global environment.

### 2.5 Constraints

The basic constraints are water cleanliness targets that are specified at the WTS levels 11 and 17 for the liquid and solid lines, respectively. Also, constraints are added on the cleanliness of the

liquid/sludge entering each level to handle a situation when liquid/sludge in the WTS are too polluted to be processed by any treatment technology units available in a particular level (as a result of empty units being selected too often in earlier levels). These range limits on the state variables entering each level are shown in Tables 2, 3, and 4. They define the state space for each level of liquid and solid lines. The lower limits are based on the highest possible pollutant removal, and the upper limits are computed based on the lowest possible pollutant removal assuming that the “empty unit” was not selected. Attainment of the constraints is achieved via a penalty function added to the objective function. The same penalty function was utilized to achieve cleanliness targets and to maintain state space limits. Mathematically, the target penalty functions are quintic functions similar to those used in Chen et al. (1999) and Tsai (2002), which comprise three knots: the lowermost ( $kn_-$ ) where there is zero penalty for being below target, the middle ( $kn$ ), and the uppermost ( $kn_+$ ). The middle and uppermost knots are defined as  $kn = kn_- + \Delta$  and  $kn_+ = kn + \Delta$ , respectively, where  $\Delta$  is determined such that the uppermost knot,  $kn_+$ , coincides with the maximum value of the effluent. The purpose of utilizing a penalty function is to assess penalty for violating liquid/sludge cleanliness targets, and the quintic form was chosen to facilitate modeling by multivariate adaptive regression splines (MARS). The cleanliness penalty is assessed in WTS level 11 for the liquid line and in WTS level 17 for the solid line. Tables 5 and 6 present the state variable ranges of the liquid/sludge exiting the wastewater treatment system, target values, cost smoothing values (denoted by  $\Delta$ ), and penalty coefficients. Target values can be easily adjusted to satisfy any desired cleanliness requirements of WTS.

## 2.6 MSMO Decision-Making Problem

The MSMO version of WTS is a continuous-state, 20-dimensional, 17-stage (levels of treatment or time periods), 6-objective optimization problem. It is formulated as a multiobjective stochastic dynamic programming, which is an extension of traditional stochastic dynamic programming (SDP) often used for the optimization of multiperiod problems in diverse areas of application such as engineering, finance, economics, etc. (Bertsekas, 2007). The multiple objective SDP can then be formulated (shown schematically in Fig. 3). The multiple objective stochastic dynamic programming formulation of WTS is

$$\begin{aligned} \text{Vmin}_{u_1, \dots, u_T} \quad & E \{ M \{ \mathbf{m}_1(\mathbf{x}_1, u_1, \boldsymbol{\epsilon}_{1,u_1}), \mathbf{m}_2(\mathbf{x}_2, u_2, \boldsymbol{\epsilon}_{2,u_2}), \dots, \mathbf{m}_T(\mathbf{x}_T, u_T, \boldsymbol{\epsilon}_{T,u_T}) \} \} \\ \text{s.t.} \quad & \mathbf{x}_{\tau+1} = f_\tau(\mathbf{x}_\tau, u_\tau, \boldsymbol{\epsilon}_{\tau,u_\tau}), \text{ for } \tau = 1, \dots, T-1, \\ & \mathbf{x}_\tau \in S_\tau, \quad u_\tau \in \Gamma_\tau, \text{ for } \tau = 1, \dots, T, \end{aligned} \tag{1}$$

where  $T$  is the time horizon,  $\mathbf{x}_\tau$  is the state vector (attributes of the liquid/sludge) at level  $\tau$ ,  $u_\tau$  is the decision vector (index of the selected technology unit) at level  $\tau$ ,  $\boldsymbol{\epsilon}_{\tau,u_\tau}$  is the random vector representing the stochastic component on the performance parameters of unit  $u_\tau$ ,  $\mathbf{x}_{\tau+1}$  is the state vector at level  $\tau + 1$  determined by the transition function  $f_\tau(\cdot)$ ,  $S_\tau$  contains the lower and upper range limits on the state variables at level  $\tau$ ,  $\Gamma_\tau$  contains the indices of the available technology units at level  $\tau$ , and the function  $M(\cdot)$  denotes the multiple objective return function. The expectation is taken over the random vector with the known probability distribution to be the objective function. For the multiple stage WTS, the expected value was estimated by generating discrete random numbers to simulate  $\boldsymbol{\epsilon}_\tau$  from a uniform distribution. The cleanliness penalty is computed in WTS level 11 for the liquid line (using the range limits in  $S_{11}$ ) and in WTS level 17

for the solid line (using the range limits in  $S_{17}$ ). Also, the objective vector at stage  $\tau$  is

$$\mathbf{m}_\tau(\mathbf{x}_\tau, u_\tau, \boldsymbol{\epsilon}_{\tau, u_\tau}) = \begin{pmatrix} m_\tau^1(\mathbf{x}_\tau, u_\tau, \boldsymbol{\epsilon}_{\tau, u_\tau}) \\ m_\tau^2(\mathbf{x}_\tau, u_\tau, \boldsymbol{\epsilon}_{\tau, u_\tau}) \\ \vdots \\ m_\tau^k(\mathbf{x}_\tau, u_\tau, \boldsymbol{\epsilon}_{\tau, u_\tau}) \end{pmatrix},$$

where  $k$  is the number of objective functions at each stage.

### 3 Approach to Solving MSMO Wastewater Treatment System

The MSMO version of WTS can be solved by combining the *multiple stage* aspect of the problem with its *multiple objective* aspect without altering the original problem. The multiple objective problems aim to determine a set of solutions representing optimal trade-offs on a set of diverse objectives (which may include conflicting and non-commensurable objectives). These are popularly referred to as Pareto optimal solutions. The methods for generating Pareto optimal points include, weighted-sum (Chankong and Haimes, 1983),  $\varepsilon$ -constraint (Chankong and Haimes, 1983), hybrid (combines both weighted-sum and  $\varepsilon$ -constraint) (Miettinen, 1999), norm or weighted metrics, min-max, etc. The weighted-sum method is used here for its applicability and the ease with which it allows decision-makers/technical experts to participate in the solution development process. The weighted-sum transforms a multiple objective optimization problem into a single objective optimization problem (schematically shown in Fig. 4). This conversion process is usually referred to as scalarization. The scalarization process results in a weighted-sum of the objective functions, which is then optimized to obtain a set of Pareto optimal solutions by varying the weights. However, a potential complication to the weighted-sum scalarization process arises due to the multiple stage aspect of the optimization problem: determining meaningful weight vectors *a priori* at each stage of a multistage problem. The three-phase methodology, discussed in (Tarun et al., 2007; Tarun, 2008), provides a systematic approach that allows decision-makers' participation in resolving the issues of determining a meaningful weight vector at one stage and modifying it appropriately to obtain the weight vectors at subsequent stages. Next, we describe the three-phase methodology and the weighted-sum scalarization of the original MSMO formulation.

#### 3.1 Three-Phase Methodology

The methodology is built upon three distinct phases. These three phases are depicted in Fig. 5 for a typical multistage and multiobjective model. This extends the multiple stage, single objective decision-making framework presented in Tsai (2002) to a multiple stage, multiple objective (MSMO) decision-making domain. The *input phase* elicits judgments from decision makers on pairs of objectives for the first stage and on dependencies from one stage to the next. The *matrix generation phase* uses experts' opinions from the input phase to construct pairwise comparison matrices for subsequent stages. The *weighting phase* uses the pairwise comparison matrices computed in the matrix generation phase to calculate weight vectors at each stage (Saaty, 1980). Subsequently, these weight vectors are used for the weighted-sum scalarization of the multistage vector optimization problem.

##### 3.1.1 The Input Phase

Let the matrices  $A_{(\tau, \tau)}$  and  $T_\tau$  be defined as follows.

- $A_{(\tau,\tau)}$  is the  $k \times k$  pairwise comparison matrix at stage  $\tau$ , where  $k$  is the number of objective functions at stage  $\tau$ , and  $a_{ij}^{(\tau,\tau)}$  is the value at the intersection of row  $i$  and column  $j$  of  $A_{(\tau,\tau)}$ .
- $T_\tau$  is the  $k \times k$  diagonal transformation matrix between stage  $\tau$  and  $\tau + 1$ , and  $t_{ij}^\tau$  is the value at the intersection of row  $i$  and column  $j$  of  $T_\tau$ .

The input phase utilizes the questionnaire approach, discussed in Tarun et al. (2008) and Tarun (2008), to obtain the following two classes of judgments from the decision makers (or experts or technical consultants) with regard to MSMO wastewater treatment system:

1. the judgment on the pairwise comparisons in the first stage to form a complete pairwise comparison matrix at stage one (denoted by matrix  $A_{(1,1)}$ )
2. the judgments on dependencies of the same classes of objective function from one stage to the next (denoted by matrices  $T_\tau$  between stage  $\tau$  and  $\tau + 1$ )

**Judgment on the Pairwise Comparisons at the First Stage:**  $A_{(1,1)}$  satisfies all the properties of the pairwise comparison matrix specified by the AHP. According to AHP, the following are the properties of a pairwise comparison matrix:

- the value in row  $i$  and column  $j$  of  $A_{(\tau,\tau)}$  (denoted by  $a_{ij}^{(\tau,\tau)}$ ) indicates how much more important objective  $i$  is than objective  $j$  at stage  $\tau$ ;
- the *importance* is measured on a ratio scale  $[\frac{1}{9}, 9]$  with each number being interpreted according to the AHP philosophy given in Saaty (1980);
- the value in row  $i$  and column  $j$  of  $A_{\tau,\tau}$  should be positive, i.e.,  $a_{ij}^{(\tau,\tau)} > 0, \forall i, j$ ;
- $a_{ii}^{(\tau,\tau)} = 1, \forall i$ ;
- for consistency it is necessary that  $a_{ji}^{(\tau,\tau)} = \frac{1}{a_{ij}^{(\tau,\tau)}}, \forall i, j$ ;
- transitivity may not hold if the decision maker is inconsistent, i.e., if  $\exists i, j, k$  such that  $[a_{ij}^{(\tau,\tau)}][a_{jk}^{(\tau,\tau)}] \neq a_{ik}^{(\tau,\tau)}$ .

We assume that there are the same  $k$  objective functions in every stage. The necessary consistency property implies a need for  $\frac{k(k-1)}{2}$  pairwise judgments in order to form a complete pairwise comparison matrix.

**Judgments on Dependencies from One Stage to the Next:** The  $k \times k$  diagonal matrix  $T_\tau$  implies a need to obtain  $k$  pairwise judgments. We assume that dependencies exist between the same objective functions in consecutive stages. This implies  $k$  pairwise judgments. Alternately, the matrix of dependencies can also be termed as an interstage diagonal transformation matrix, named from the role it plays in transforming the pairwise comparison matrix in one stage into the pairwise comparison matrix in next stage following the methodologies for matrix generation in the second phase. The properties of the matrix of dependencies (or interstage diagonal transformation matrix),  $T_\tau$ , are:

- the value in row  $i$  and column  $i$  of  $T_\tau$  (denoted by  $t_{ii}^\tau$ ) indicates how much more important objective  $i$  in the stage  $\tau$  is than objective  $i$  in the stage  $\tau + 1$ ;

- the importance is measured on a ratio scale  $[\frac{1}{9}, 9]$  with each number being interpreted according to AHP philosophy given in Saaty (1980);
- the value of non-diagonal elements of  $T_\tau$  should be zero, i.e.,  $t_{ij}^\tau = 0$ , for  $i \neq j$ ;
- diagonal elements of  $T_\tau$  should be positive, and belong to the AHP ratio scale  $[\frac{1}{9}, 9]$ , i.e.,  $t_{ii}^\tau > 0$  and  $t_{ii}^\tau \in [\frac{1}{9}, 9]$ .

The questionnaire modeling of the input phase is unique in the way it is applied to obtain pairwise judgments across various stages in a multiple stage, multiple objective problem. The input phase, with the exception of matrix of dependencies from one stage to the next or interstage diagonal transformation matrix (Tarun et al., 2007; Tarun, 2008), follows the standard AHP approach (Hobbs and Meier, 2000). The implementation results of questionnaire modeling for the 17-stage, 6-objective wastewater treatment system are shown in the next section.

### 3.1.2 The Matrix Generation Phase

This step is crucial for achieving the primary objective of weight vector generation. In this phase, pairwise comparison matrices are computed for all stages. Our primary motivation for the new methods was to attain pairwise comparison matrices that comply with the AHP ratio scale. These methods are:

1. Geometric mean (GM)
2. Successive geometric mean (SGM).

Aside from satisfying the AHP ratio scale, GM and SGM approaches do not distort the scaling. We first define the new functions  $g_p$  and  $G$  that form the basis for both GM and SGM methods.

**Function Definitions:** Let  $\nu_p$  be a function such that  $\nu_p : \mathbb{R}^p \rightarrow \mathbb{N}$  where  $\mathbb{R}$  is the set of all real numbers on the AHP ratio scale  $[\frac{1}{9}, 9]$ ,  $p$  is the number of input matrices,  $\mathbb{N}$  is the set of all natural numbers less than or equal to  $p$ , and

$$\nu_p(\alpha_1, \alpha_2, \dots, \alpha_p) = \text{number of non-one } \alpha_i\text{'s, if } \exists \alpha_i \neq 1 \text{ for some } i = 1, 2, \dots, p. \quad (2)$$

Then let  $g_p$  be a function such that  $g_p : \mathbb{R}^p \rightarrow \mathbb{R}$ , and

$$\begin{aligned} g_p(\alpha_1, \alpha_2, \dots, \alpha_p) &= (\alpha_1 \alpha_2 \dots \alpha_p)^{\frac{1}{\nu_p(\alpha_1, \alpha_2, \dots, \alpha_p)}}, \text{ if } \exists \alpha_i \neq 1 \text{ for some } i = 1, 2, \dots, p, \\ &= 1, \text{ otherwise.} \end{aligned} \quad (3)$$

Let there be a set of  $p \times k$  matrices where  $p$  is an odd number greater than or equal to 3. Of the  $p$  matrices let there be  $p - 1$  diagonal matrices  $D_q = (d_{ij}^q)$ ,  $q = 1, 2, \dots, (p - 1)$  with positive diagonal entries  $d_{ii}^q > 0$  for  $i = 1, 2, \dots, k$ , and  $\Theta = (\theta_{ij})$  being a real matrix with positive entries  $\theta_{ij} > 0$  for  $i, j = 1, 2, \dots, k$ . Then let us define a function  $G$  such that

$$G(D_1, D_2, \dots, D_{(\frac{p-1}{2})}, \Theta, D_{(\frac{p-1}{2}+1)}, D_{(\frac{p-1}{2}+2)}, \dots, D_{(p-1)}) = \Theta', \quad (4)$$

where  $\Theta' = (\theta'_{ij})$  is a  $k \times k$  matrix, and

$$\theta'_{ij} = g_p(d_{ii}^1, d_{ii}^2, \dots, d_{ii}^{(\frac{p-1}{2})}, \theta_{ij}, d_{jj}^{(\frac{p-1}{2}+1)}, d_{jj}^{(\frac{p-1}{2}+2)}, \dots, d_{jj}^{(p-1)}). \quad (5)$$

We next describe the computation of pairwise comparison matrices using GM and SGM methods.

**Geometric Mean (GM):** The GM method calculates the *geometric mean of non-ones*: the first iteration computes a matrix containing the geometric mean of non-ones of the multiplication of matrices  $(T_1)^{-1}$ ,  $A_{(1,1)}$ , and  $T_1$ ; the second iteration computes a matrix containing the geometric mean of non-ones of the multiplication of  $(T_2)^{-1}$ , three matrices from the first iteration, and  $T_2$ ; and an arbitrary  $(\tau-1)$ th iteration computes the geometric mean of non-ones of the multiplication of  $(T_{\tau-1})^{-1}$ ,  $2\tau-3$  matrices from  $(\tau-2)$ th iteration, and  $T_{\tau-1}$ . The meaning of the geometric mean of non-ones becomes clear from the second iteration onward. It involves computation of a matrix containing the geometric mean of non-ones of the multiplication of  $2i+1$  matrices, where  $i$  is the iteration. Given the matrices from the input phase and using the definition of  $G(\cdot)$  from the section on function definitions with  $p=2i+1$ , the GM computation is as follows:

**1st iteration:** Pairwise comparison matrix at stage 2

$$A_{(2,2)} = G[(T_1)^{-1}, A_{(1,1)}, T_1].$$

**2nd iteration:** Pairwise comparison matrix at stage 3

$$A_{(3,3)} = G[(T_2)^{-1}, (T_1)^{-1}, A_{(1,1)}, T_1, T_2].$$

⋮

**$(\tau-1)$ st iteration:** Pairwise comparison matrix at stage  $\tau$

$$A_{(\tau,\tau)} = G[(T_{\tau-1})^{-1}, (T_{\tau-2})^{-1}, \dots, (T_1)^{-1}, A_{(1,1)}, T_1, T_2, \dots, T_{\tau-1}].$$

For  $i > j$  at an arbitrary stage  $\tau$ , values in the pairwise comparison matrix  $A_{(\tau,\tau)}$  can be expressed as:

$$a_{ij}^{(\tau,\tau)} = \left[ \left( \left( \frac{t_{jj}^{\tau-1}}{t_{ii}^{\tau-1}} \right) \left( \frac{t_{jj}^{\tau-2}}{t_{ii}^{\tau-2}} \right) \dots \left( \frac{t_{jj}^1}{t_{ii}^1} \right) \right)^{\frac{1}{N_{ij}^\tau}} \right] \left[ (a_{ij}^{(1,1)})^{\frac{1}{N_{ij}^\tau}} \right], \quad (6)$$

for  $\tau = 2, 3, \dots, T$ , where  $N_{ij}^\tau$  is the number of non-ones involved in the GM computation of  $a_{ij}^{(\tau,\tau)}$ .

**Successive Geometric Mean (SGM):** The SGM method calculates the *geometric mean of non-ones successively*: the first iteration computes a matrix containing the geometric mean of non-ones of the multiplication of matrices  $(T_1)^{-1}$ ,  $A_{(1,1)}$ , and  $T_1$ ; the second iteration computes a matrix containing the geometric mean of non-ones of the multiplication of  $(T_2)^{-1}$ , the resulting matrix from the first iteration, and  $T_2$ ; etc. Again, the meaning of the successive geometric mean of non-ones becomes clear from the second iteration onward. It involves computation of a matrix containing the geometric mean of non-ones of the multiplication of three matrices, one of which is a matrix having geometric mean of non-ones from the previous iteration. Given the matrices from the input phase and using the definition of  $G(\cdot)$  from the section on function definitions with  $p=3$  for all iterations, the SGM computation becomes:

**1st iteration:** Pairwise comparison matrix at stage 2

$$A_{(2,2)} = G[(T_1)^{-1}, A_{(1,1)}, T_1].$$

**2nd iteration:** Pairwise comparison matrix at stage 3

$$A_{(3,3)} = G[(T_2)^{-1}, A_{(2,2)}, T_2].$$

⋮

**( $\tau-1$ )st iteration:** Pairwise comparison matrix at stage  $\tau$

$$A_{(\tau,\tau)} = G[(T_{\tau-1})^{-1}, A_{(\tau-1,\tau-1)}, T_{\tau-1}].$$

For  $i > j$  at an arbitrary stage  $\tau$ , values in the pairwise comparison matrix  $A_{(\tau,\tau)}$  can be expressed as:

$$a_{ij}^{(\tau,\tau)} = \left[ \left( \frac{t_{jj}^{\tau-1}}{t_{ii}^{\tau-1}} \right)^{\frac{1}{N_{ij}^\tau}} \left( \frac{t_{jj}^{\tau-2}}{t_{ii}^{\tau-2}} \right)^{\frac{1}{N_{ij}^\tau N_{ij}^{\tau-1}}} \dots \left( \frac{t_{jj}^1}{t_{ii}^1} \right)^{\frac{1}{N_{ij}^\tau N_{ij}^{\tau-1} \dots N_{ij}^2}} \right] \left[ (a_{ij}^{(1,1)})^{\frac{1}{N_{ij}^\tau N_{ij}^{\tau-1} \dots N_{ij}^2}} \right], \quad (7)$$

for  $\tau = 2, 3, \dots, T$ , where  $N_{ij}^\tau$  is the number of non-ones involved in the SGM computation of  $a_{ij}^{(\tau,\tau)}$ .

The geometric mean-based methods for computing pairwise comparison matrices, GM and SGM, are unique not only in the way they reduce the time to generate pairwise comparison matrices but also in the fashion they help maintain the component values of resulting pairwise comparison matrices in the AHP ratio scale range. Further, the geometric-mean based methods produce highly consistent pairwise comparison matrices, as demonstrated in Tarun (2008).

### 3.1.3 The Weighting Phase

The weighting phase is identical for both GM and SGM methods of pairwise comparison matrix generation. Saaty's eigenvector method is used to approximate the principal eigenvector associated with each pairwise comparison matrix. These principal eigenvectors are used as the weight vectors. The weight vector calculation procedure requires one to normalize the pairwise comparison matrix  $A_{(\tau,\tau)}$  at a stage  $\tau$  by dividing each entry in column  $j$  by the sum of entries in column  $j$ , which is denoted by  $A_{(\tau,\tau)}^{norm}$  and approximate the principal eigenvector (termed as weight vector  $W_\tau$  at stage  $\tau$ ) by finding the average of each row of the normalized matrix. Consistent pairwise comparison matrices result in meaningful weight vectors to be used for weighted-sum scalarization. Implementation results of weighting phase are shown in the next section.

One of the major advantages of the three-phase methodology is the significant reduction in the amount of information required from the experts/decision makers in the input phase. Since all pairwise comparisons are considered only for the first stage, and the pairwise comparison matrices for the subsequent stages are computed based on comparing only  $k$  pairwise objectives instead of  $\frac{k(k-1)}{2}$ , where  $k$  is the number of objectives at each stage. Mathematically,  $k < \frac{k(k-1)}{2}$  for  $k > 3$ .

The three-phase methodology extends analytic hierarchy process (AHP) to the MSMO decision-making problems along with the following features: high interpretability as pairwise comparison matrices maintain the component values in the AHP ratio scale range; highly consistent pairwise comparison matrices at each stage due to interactions with the experts; actual decision makers expressing their judgments on the relative importance of objectives at a stage and between two consecutive stages without being overwhelmed with the technical details; and generation of meaningful weight vectors for the scalarization of the MSMO version of WTS.

## 3.2 Weighted-Sum Scalarization

The next task is to generate Pareto optimal solutions. Some of the methods for generating Pareto optimal points include, weighted-sum (Chankong and Haimes, 1983),  $\varepsilon$ -constraint (Chankong and Haimes, 1983), hybrid that combines both weighted-sum and  $\varepsilon$ -constraint (Miettinen, 1999), norm or weighted metrics, and minimax. The weighted-sum method is used here for its ease of application and involvement of decision makers in the solution process. The weighted-sum transforms multiple objective functions into a single objective function, and optimizes the weighted-sum of

the objectives. This conversion process is usually referred to as weighted-sum scalarization. The weighted-sum scalarized form of the problem (shown schematically in Fig. 4) becomes

$$\begin{aligned} \min_{u_1, \dots, u_T} \quad & E \left\{ \sum_{\tau=1}^T \mathbf{W}_\tau \mathbf{m}_\tau(\mathbf{x}_\tau, u_\tau, \boldsymbol{\epsilon}_{\tau, u_\tau}) \right\} \\ \text{s.t.} \quad & \mathbf{x}_{\tau+1} = f_\tau(\mathbf{x}_\tau, u_\tau, \boldsymbol{\epsilon}_{\tau, u_\tau}), \text{ for } \tau = 1, \dots, T-1, \\ & \mathbf{x}_\tau \in S_\tau, \quad u_\tau \in \Gamma_\tau, \text{ for } \tau = 1, \dots, T, \end{aligned} \quad (8)$$

where  $\mathbf{W}_\tau$  is the *weight vector* at stage  $\tau$ ,  $(w_\tau^1, w_\tau^2, \dots, w_\tau^k)$ , and  $\mathbf{m}_\tau(\mathbf{x}_\tau, u_\tau, \boldsymbol{\epsilon}_{\tau, u_\tau})$  is the *objective vector* at stage  $\tau$ ,  $(m_\tau^1(\mathbf{x}_\tau, u_\tau, \boldsymbol{\epsilon}_{\tau, u_\tau}), m_\tau^2(\mathbf{x}_\tau, u_\tau, \boldsymbol{\epsilon}_{\tau, u_\tau}), \dots, m_\tau^k(\mathbf{x}_\tau, u_\tau, \boldsymbol{\epsilon}_{\tau, u_\tau}))'$ . The equation 8 is the resulting multiple stage, single objective optimization problem.

### 3.3 Normalization/transformation of objective functions

It is practical to transform or normalize the objective functions so that they all have comparable orders of magnitude. We use the *upper-lower-bound approach* recommended in Marler and Arora (2005). For a given stage  $\tau$ , this approach uses the transformation,

$$mt^i = \frac{m^i(\mathbf{u}) - m_0^i}{m_{\max}^i - m_0^i}, \quad (9)$$

$$m_{\max}^i = \max_{1 \leq j \leq k} m^i(\mathbf{u}_j^*), \quad (10)$$

for  $i = 1, \dots, k$  where  $k$  is the number of objective functions and  $\mathbf{u}_j^*$  is the point that minimizes the  $j$ th objective function,

$$m_0^i = \min_{\mathbf{u}} \{m^i(\mathbf{u}) | \mathbf{u} \in \Gamma\}, \quad (11)$$

where  $\Gamma$  is the feasible decision space.

Therefore, the normalized/transformed form of the multiple stage, single objective problem in equation 8 is,

$$\begin{aligned} \min_{u_1, \dots, u_T} \quad & E \left\{ \sum_{\tau=1}^T (\mathbf{W}_\tau \star \mathbf{MT}_\tau) \mathbf{m}_\tau(\mathbf{x}_\tau, u_\tau, \boldsymbol{\epsilon}_{\tau, u_\tau}) \right\} \\ \text{s.t.} \quad & \mathbf{x}_{\tau+1} = f_\tau(\mathbf{x}_\tau, u_\tau, \boldsymbol{\epsilon}_{\tau, u_\tau}), \text{ for } \tau = 1, \dots, T-1, \\ & \mathbf{x}_\tau \in S_\tau, \quad u_\tau \in \Gamma_\tau, \text{ for } \tau = 1, \dots, T, \end{aligned} \quad (12)$$

where  $\star$  is the symbol for component-wise vector multiplication,  $\mathbf{W}_\tau$  is the *weight vector* at stage  $\tau$ ,  $(w_\tau^1, w_\tau^2, \dots, w_\tau^k)$ , and  $\mathbf{MT}_\tau$  is the *objective transformation vector* at stage  $\tau$ ,  $(mt_\tau^1, mt_\tau^2, \dots, mt_\tau^k)$ . Therefore, the problem that needs to be solved eventually is the normalized multiple stage, single objective problem formulation described above in equation 12. The solution approach to the resulting optimization model is described next.

### 3.4 Solution to Normalized Multiple Stage, Single Objective Optimization Problem

The normalized multiple stage, single objective problem formulation described above in equation 12 is solved using an approach that augments the high-dimensional, continuous-state stochastic dynamic programming method described in Tsai (2002) and Chen et al. (1999) to deal with multiple objectives at each stage. In particular, the following modifications were made to address the present needs.

- A routine was added to capture decision makers' preferences, which get translated into weight vectors using the three-phase methodology.
- A routine was added to normalize/transform the objective functions with different units to have similar units and orders of magnitude.
- State transition equations were added for new technology units- Yellow Water Separation and Yellow and Black Water Separation in level 1, and UASB System *plus* Activated Sludge Process (C, P, N) in level 5.
- Constraints (both liquid and sludge state variable limits in each level) were modified to reflect the addition of new technology units.
- Penalty coefficients and costsmoother parameters were modified due to addition of new technologies.

Some of the issues with the augmented stochastic dynamic programming model include presence of continuous-state variables, presence of uncertainty, 20-dimensional state vector, form of state transitions, and future value function approximation. To resolve the continuous nature of state variables, the orthogonal array (OA) based Latin hypercube designs, presented in (Chen et al., 1999; Chen, 2001), are used for state space discretization, and the MARS (Tsai and Chen, 2005) algorithm is used for future value function approximation. The details on the orthogonal array based Latin hypercubes and MARS algorithm can be seen in Tsai (2002).

A small and a big design with 2209 and 12167 discretization points, respectively, are considered for demonstrating the results from implementation of the augmented stochastic dynamic programming approach to evaluate wastewater treatment technologies. Next, a brief explanation is given for how these design points are used to evaluate technology processes/units in wastewater treatment system (WTS).

The future value functions are obtained backward. Starting the iteration in the last level, for each discretization/design point in the last level we solve the optimization problem that minimizes the normalized weighted-sum of objective functions. Then, we approximate the solution to this minimization problem through a future value function, for all the discretization points, obtained by fitting a statistical model to the data obtained in the previous step. This results in the selection of optimal technologies for each of the design points in the last level. Moving to the next level (last but one), for each discretization point we solve the optimization problem that minimizes the (*the weighted-sum of objective functions in the current level + the future value function obtained from the last level*). Then we approximate the solution to optimization problem through a future value function, for all the discretization points in the current level, obtained by fitting a statistical model to the data obtained in the previous step. This results in the selection of optimal technologies for each of the design points in the current level. The process is repeated until the future value function for the level 1 is obtained.

## 4 Implementations and Results

### 4.1 Three-Phase Methodology

We are applying the three-phase methodology (Tarun et al., 2007) on a conceptual multiple stage, multiple objective wastewater treatment system. Table 1 shows wastewater treatment technology choices at all levels of WTS. The stages correspond to different levels of processing in WTS. The MSMO version of WTS has two new technologies at level 1 and one new technology at level 5 of the

liquid processing line. Again, the goal is to select the treatment technology units at each level of WTS for *minimizing* economic cost, size, and odor emissions, while *maximizing* nutrient recovery, robustness, and global desirability. Results of the three-phase implementation are presented next.

#### 4.1.1 Input phase

The questionnaire modeling, discussed in Tarun et al. (2008), is used in the input phase to elicit decision makers' preferences on the tradeoffs between all possible pairs of objectives at a stage and between same objective types in consecutive stages. A single technology unit "empty unit" at level 4 of the wastewater treatment system (WTS) implies that no optimization is needed at this level, which means there is no need of a questionnaire at stage 4 (or level 4). In other words, the questionnaire-modeling involves tradeoff questions at levels 1 through 3, and 5 through 17 of the WTS. We obtained answers to a total of 390 questions in various questionnaires from Georgia Department of Natural Resources (GADNR). We denote the WTS objectives at level  $\tau$  as follows.

- $EC_\tau$  denotes the Economic Cost (in *USD*).
- $S_\tau$  denotes the Land Area (in  $m^2$ ).
- $O_\tau$  denotes the Odor Emissions (in *mg/min*).
- $NR_\tau$  denotes the Nutrient Recovery (on 1-5 scale).
- $R_\tau$  denotes the Robustness (no units).
- $GD_\tau$  denotes the Global Desirability (on 1-6 scale).

The questionnaire had a set of questions on the worst/best values for the objectives at each level, the relative importance for different objectives at level 1, and the relative importance for same types of objectives across different levels of the WTS. The WTS questionnaire is organized in the following tables.

- Table 7 presents the worst and the best values for all six objectives at levels 1 through 3. These values are based on the WTS code developed by Jining Chen (Chen, 1993; Chen and Beck, 1997). The worst and best values for all six objectives at all 17 levels can be seen in Tarun (2008).
- Table 8 presents importance questions for comparing the different objectives within Level 1 of WTS.
- Table 9 presents interlevel importance questions for comparing the same objective types across different levels of WTS. The complete tables for all six objectives can be seen in Tarun (2008).

The worst and best values in Table 7 help decision makers answer tradeoff questions (or relative importance questions) in Tables 8 and 9 with a relatively high level of precision and consistency. We use Tables 8 and 9 to get the complete pairwise comparison matrix at level 1 and interstage diagonal matrices, respectively.

The questionnaire-based approach results in the pairwise comparison matrix at stage 1,  $A_{(1,1)}$ , and interstage diagonal transformation matrices (or matrices of dependencies from one stage to

the next),  $T_\tau$ , for  $\tau = 1, 2, \dots, (T-1)$ , where  $T=17$ . These matrices are utilized as inputs to the matrix generation phase. The resulting pairwise comparison matrix at stage 1,

$$A_{(1,1)} = \begin{matrix} & \mathbf{R} & \mathbf{S} & \mathbf{O} & \mathbf{GD} & \mathbf{EC} & \mathbf{NR} \\ \mathbf{R} & 1 & 3 & \frac{1}{4} & \frac{1}{2} & 4 & 4 \\ \mathbf{S} & \frac{1}{3} & 1 & \frac{1}{6} & \frac{1}{4} & 2 & \frac{1}{2} \\ \mathbf{O} & 4 & 6 & 1 & 2 & 8 & 7 \\ \mathbf{GD} & 2 & 4 & \frac{1}{2} & 1 & 6 & 6 \\ \mathbf{EC} & \frac{1}{4} & \frac{1}{2} & \frac{1}{8} & \frac{1}{6} & 1 & \frac{1}{3} \\ \mathbf{NR} & \frac{1}{4} & 2 & \frac{1}{7} & \frac{1}{6} & 3 & 1 \end{matrix}.$$

The sixteen interstage diagonal transformation matrices are,

$$\begin{aligned} T_1 &= \begin{pmatrix} \frac{1}{7} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{8} & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix}, T_2 = \begin{pmatrix} \frac{1}{5} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{8} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{9} \end{pmatrix}, T_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \\ T_4 &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, T_5 = \begin{pmatrix} \frac{1}{5} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{8} & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix}, T_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \\ T_7 &= \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{5} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{9} \end{pmatrix}, T_8 = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}, T_9 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{9} \end{pmatrix}, \\ T_{10} &= \begin{pmatrix} 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9 \end{pmatrix}, T_{11} = \begin{pmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, T_{12} = \begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \\ T_{13} &= \begin{pmatrix} \frac{1}{8} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{8} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{8} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}, T_{14} = \begin{pmatrix} 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \end{aligned}$$

$$T_{15} = \begin{pmatrix} \frac{1}{9} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{9} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{9} & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{9} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{9} \end{pmatrix}, T_{16} = \begin{pmatrix} 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9 \end{pmatrix}.$$

The above matrices are used as inputs to the matrix generation phase to generate pairwise comparison matrices using geometric-mean based methods, GM and SGM, at each stage of the MSMO version of the WTS. The results for matrix generation phase are presented next.

#### 4.1.2 Matrix generation phase

For both GM and SGM methods, the computed pairwise comparison matrix at stage 2 is,

$$A_{(2,2)} = \begin{matrix} & \mathbf{R} & \mathbf{S} & \mathbf{O} & \mathbf{GD} & \mathbf{EC} & \mathbf{NR} \\ \mathbf{R} & \begin{pmatrix} 1.000000 & 1.912931 & 0.704730 & 1.052727 & 1.518294 & 4.820285 \end{pmatrix} \\ \mathbf{S} & \begin{pmatrix} 0.522758 & 1.000000 & 0.464159 & 0.629961 & 0.908560 & 1.817121 \end{pmatrix} \\ \mathbf{O} & \begin{pmatrix} 1.418983 & 2.154435 & 1.000000 & 1.493802 & 1.709976 & 5.192494 \end{pmatrix} \\ \mathbf{GD} & \begin{pmatrix} 0.949914 & 1.587401 & 0.669433 & 1.000000 & 1.310371 & 4.160168 \end{pmatrix} \\ \mathbf{EC} & \begin{pmatrix} 0.658634 & 1.100642 & 0.584804 & 0.763143 & 1.000000 & 2.201285 \end{pmatrix} \\ \mathbf{NR} & \begin{pmatrix} 0.207457 & 0.550321 & 0.192586 & 0.240375 & 0.454280 & 1.000000 \end{pmatrix} \end{matrix}.$$

The computed pairwise comparison matrices for WTS levels 3 through 17 using the methods, geometric mean (GM) of non-ones and successive geometric mean (SGM) of non-ones, for matrix generation can be seen in Tarun (2008).

#### 4.1.3 Weighting phase

Saaty's eigenvector approach (Saaty, 1980) is used to compute weight vectors for both GM and SGM methods in the matrix generation phase. These are principal eigenvectors for the pairwise comparison matrices obtained in the matrix generation phase. Tables 10 and 11 are the weight vectors obtained using the GM and SGM methods, respectively.

### 4.2 Computational results on the consistency of computed pairwise matrices

Lower values of consistency indices indicate better consistencies of corresponding pairwise comparison matrices (Saaty, 1980). These results show that the pairwise comparison matrices computed in the matrix generation phase are highly consistent implying that the corresponding weight vectors computed in the weighting phase are meaningful. Therefore the weight vectors can be used to scalarize the MSMO version of the WTS. Highly consistent pairwise comparison matrices imply high level of consistency in the decision makers' judgments elicited in the input phase.

For GM method, consistency indices decrease from one stage to the next as seen in Table 12 and Fig. 6. Further, the consistency indices for the SGM method are higher than the indices for the GM method implying that the GM method performs better in terms of consistency of judgments elicited from decision makers.

### 4.3 Solution Results for the Scalarized Version of WTS

Since the weight vectors obtained were based on highly consistent pairwise comparison matrices, they can be used for scalarization of the MSMO version of WTS using weighted-sum of objective functions at each stage. The scalarization converts the MSMO decision-making problem into a multiple stage, single objective decision-making problem, which is also referred to as stochastic dynamic programming (SDP) formulation. We next present the results obtained using the augmented stochastic dynamic programming solution approach to solve the scalarized version of WTS.

#### 4.3.1 Results for the small design

Tables 13 and 14 present the resulting counts for a solution with the GM and the SGM methods respectively, using an orthogonal array and Latin hypercube based experimental design (OA-LHD) with  $N=2209$  generated from a 47-level strength two orthogonal array. In the results table, “**bold-face**” signifies technology units that are new (may only be existing as prototypes) or new for a specific level, and “*italics*” signifies somewhat new technology units (may not be well understood yet). In order for MARS to estimate the future value functions, the maximum number of basis functions and number of eligible knots considered are 200 and 35, respectively. The dependencies between technology units are evident in Tables 13 and 14. For instance, the technology unit *Sludge Thickening Tank* gets picked most of the time in level 12 and *Aerobic-Anaerobic Digestion* quite a significant number of times in level 13, thereby enabling the selection of *Filter and Belt* in level 14.

Yellow Water Separation, the new technology in level 1, is a clear winner for both GM and SGM. Vortex SSO and Chemical Precipitation look promising in level 2. Ozonation and Physical Irradiation are picked up in level 3. In level 5, A-B System is only selected using the GM method while Activated Sludge (C, P, N) is selected only with the SGM method. The use of a technology is necessary for levels 6 and 7. Both Physical Irradiation and Ozonation appear to be promising in level 8. Air Stripping gets selected more often in level 9. None of the technology units in level 10 appears to be a good candidate. GAC Adsorption is a clear winner in level 11. In the solid line, Sludge Thickening Tank in level 12 appears to be highly promising. Sludge Dewatering Bed and Aerobic-Anaerobic Digestion are effective in level 13. In level 14, Filter and Belt is favored in counts. However, Permanent Thermal Process and Thermo-Chemical Liquefaction look promising as well. Sludge Dewatering Bed and Physical Irradiation work well in levels 15 and 16 respectively. In level 17, Chemical Fixation appears to win by count with GM while settling for a second place with SGM.

#### 4.3.2 Results for the big design

To validate the results using the small design, a solution was obtained using an OA-LHD with 12167 design points generated from a 23-level strength three orthogonal array. The results, which should be more precise than the small design, can be seen in Tables 15 and 16. The coefficients of multiple determination ( $R^2$ ) are shown inside the square brackets. The dependencies between technology units can be easily seen.

In level 1, Yellow Water Separation is a clear winner for both GM and SGM. In level 2, Vortex SSO is a winner by count with GM, and is a clear winner with SGM. Ozonation is a clear winner in level 3. For GM, both Multi-reactor/Deep and UASB+Activated Sludge (C, P, N) appear to be promising in level 5. However, for SGM UASB+Activated Sludge (C, P, N) looks to be a heavy favorite. Microfiltration and Secondary Settler appear to have potential in level 6, while Reverse Osmosis and Microfiltration are shown to be effective in level 7. Ozonation is dominant in level 8. Air Stripping outshines other units in level 9. Results for levels 10 and 11 are identical to the small

design. Results for the solid line look similar to the results using the small design.

### 4.3.3 Discussions

We now compare the results for the big design with that of the small design. We discuss first the results using GM method, and then those for the SGM method. In the GM method results, Yellow Water Separation wins clearly for both the designs in level 1. It is interesting to see the newly-added Yellow Water Separation unit get selected in level 1. In level 2, the small design selects Vortex SSO and Chemical Precipitation while the big design also selects Sedimentation Tank and Empty Unit. Nonetheless, there is a consistency in the selection of Vortex SSO as a heavy favorite in level 2. In level 3, Ozonation wins the selection on count for both the designs. Levels 5, 6, 7, and 8 have the following interesting observations.

- In level 5, Multi-reactor/Deep emerges as another legitimate option for the big design in addition to UASB + Activated Sludge (C, P, N). It is encouraging to see UASB + Activated Sludge (C, P, N), the newly-added unit in level 5, as the most promising technology.
- In level 5, UASB System gets selected for the big design as opposed to the small design.
- In level 5, The set of technological units selected for the big design is smaller in comparison with the small design.
- Results for levels 6 and 7 confirm a necessity for the use of technological units.
- Level 6 for the big design selects Microfiltration as the winner followed by Secondary Settler, Chemical Precipitation, and Reverse Osmosis. On the other hand, level 6 for the small design declares Chemical Precipitation as a clear winner on count while selecting other technology units occasionally.
- In level 7, both big and small design concur on Reverse Osmosis as the major selection. However, the big design has a better balance in terms of how often other technology units get selected. Microfiltration and Chemical Precipitation are selected frequently for the big design, while Physical Filtration, Microfiltration, and Chemical Precipitation are the choices for the small design.
- In level 8, the big design selects Ozonation as the winner on count followed by Physical Irradiation, which swaps the results for small design interestingly.

Level 9 selects Air Stripping far more often for both the designs. It is apparent that level 10 does not require the use of a technology unit. In level 11, GAC Adsorption is clearly superior. In the solid line, the results are more or less consistent for both the designs.

In the SGM method results, Yellow Water Separation wins clearly for both small and big designs in level 1. The big design clearly declares Vortex SSO and Ozonation as winners in levels 2 and 3 respectively. The small design, however, also selects Chemical Precipitation in level 2 and Physical Irradiation (rather infrequently) in level 3. For both the designs, level 5 selects UASB + Activated Sludge (C, P, N) to be the winner. Interestingly, the small design selects Activated Sludge (C, N), Activated Sludge (C, P, N), and Activated Sludge (N) in level 5, which do not get picked up at all for the big design. Trends similar to the GM method can be seen in levels 6 through 17.

In summary, both the methods show the new technologies in levels 1 and 5 to be promising. This observation justifies the decision to include these technologies in the evaluation process. Also, the solution obtained satisfies the dependencies between technology units.

## 5 Concluding Remarks

This paper addresses the disconnect between the decision makers and the solution developers for an MSMO optimization problem through the use of a questionnaire approach that allows decision-makers to participate in the solution development process. Other contributions of this paper are summarized next. First, it extends AHP approach to a multiple-stage decision-making framework using three-phase methodology. Second, it helps develop theories and methodologies for computing pairwise comparison matrices that have traditionally been constructed using direct inputs from decision makers. Computation of pairwise comparison matrices reduces the amount of information required from the decision makers, thereby increasing efficiency of the solution process. More importantly, the new methods in the matrix generation phase, GM and SGM, result in highly consistent pairwise comparison matrices. Third, we modify the high-dimensional, continuous-state stochastic dynamic programming (SDP) approach in Tsai (2002) to handle multiple objectives by augmenting it with two new routines: first, the routine that allows inputs from decision makers in form of weight vectors at each stage; and second, the routine that normalizes/transforms the objective functions with different units and orders of magnitude that is crucial to a practical application of weighted-sum of objective functions approach. Finally, we solve a 20-dimensional, 17-stage, 6-objective, continuous-state wastewater treatment system (WTS), which is larger than any numerically solved problem in the current literature.

We use our three-phase methodology with the augmented high-dimensional, continuous-state SDP to solve an MSMO optimization model. This technique is quite *general* in its application to a variety of large-scale MSMO problems. Also, the technique is *pragmatic* in the sense of involving decision makers in the solution development process. We have demonstrated it on a 20-dimensional, 17-stage, 6-objective, continuous-state wastewater treatment system (WTS). The solution obtained satisfies all the constraints and complications such as dependencies between technologies, etc. The final solution to WTS selects the new technologies, Yellow Water Separation and UASB System *plus* Activated Sludge Process (C, P, N), in levels 1 and 5, respectively. This result justifies the experts' decision to include these new technologies for the evaluation purposes. However, our three-phase methodology might benefit from the following refinements:

- Exploring the involvement of multiple decision makers in order to achieve a desired level of objectivity in the *Input phase*,
- Developing a theoretical basis for the consistency behavior of computed pairwise comparison matrices at a stage using the GM and SGM methods in the *Matrix Generation Phase*,
- Exploring alternate weight determination methods in the *Weighting Phase* in a quest to get a better weight estimate.

The results for consistency index indicate that the consistency indices decrease along the stages (or levels), implying that the judgments seem to improve as we move from one stage to the next for GM method. However, the consistency results are not as conclusive for SGM method. A theoretical method for comparing the results should be developed. The idea is to construct a mathematical proof to show that the consistency indices for the computed pairwise comparison matrices using the GM and the SGM methods, in general, decrease in value while moving forward across the stages. It is based on the AHP-based fact that a lower value of consistency index indicates a more consistent judgment from the decision maker.

In addition, there is a need to implement alternate methods for weight determination in the weighting phase of the three-phase methodology. It is possible that these methods could provide

weight estimates better than the Saaty's eigenvector method. Some of the promising weight-determination methods are: the alternative eigenvector method by Cogger and Yu (1985), the graded eigenvector method (GEM) of Takeda et al. (1987), the three methods for weight derivation based on pairwise comparison matrices of Krovak (1987), and the weight determination based on the decision makers' qualitative information by Batishchev et al. (1991).

Group decision making is a central feature of today's organizational decision making. The input phase in our three-phase methodology depends on the responses from single expert thereby making it relatively subjective. If the questionnaire could account for responses from multiple decision makers, the input phase could be improved in terms of consistency, accuracy, and representation of experts' preferences. In other words, the goal would be to collect inputs from multiple decision makers at a time in order to achieve some level of objectivity in the judgment.

Currently, our three-phase methodology is based on inputs from one decision maker thereby making it more or less deterministic. Though AHP has its value in terms of maintaining and ensuring consistency in decision makers' judgments/preferences, nonetheless it could do much better if the pairwise comparison matrices were stochastic reflecting uncertainties or subjectivities in human judgments. In addition to asking the decision makers *a priori* about their preferences, we could also investigate the possibility of presenting them with multiple Pareto optimal solutions for multiple sets of stagewise weight vectors. In solving weighted-sum version of the stochastic dynamic programming, we could also investigate various experimental designs for the discretization of state space and other statistical modeling approaches for the future value approximation.

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Table 1: Decision variables (or treatment technologies) at each level of the wastewater treatment system.

LIQUID LINE OF WTS						
Level	1	2	3	4	5	
Technology Units	<ul style="list-style-type: none"> <li>Flow Equalization Tank</li> <li>Yellow Water Separation</li> <li>Yellow &amp; Black Water Separation</li> </ul>	<ul style="list-style-type: none"> <li>Empty Unit</li> <li>Vortex SSO</li> <li>Sedimentation Tank</li> <li>Chemical Precipitation</li> </ul>	<ul style="list-style-type: none"> <li>Empty Unit</li> <li>Physical Irradiation</li> <li>Ozonation</li> </ul>	<ul style="list-style-type: none"> <li>Empty Unit</li> </ul>	<ul style="list-style-type: none"> <li>Empty Unit</li> <li>Activated Sludge (C)</li> <li>Activated Sludge (C, N)</li> <li>Activated Sludge (C, P)</li> <li>Activated Sludge (C, P, N)</li> <li>High Biomass Act. Sludge (C, N)</li> <li>Activated Sludge (N)</li> <li>Multi-reactor/Deep</li> </ul>	<ul style="list-style-type: none"> <li>A-B System</li> <li>Trickling Filter</li> <li>Rotating Biological Contractors</li> <li>UASB System</li> <li>UASB System + Activated Sludge (C, P, N)</li> <li>Reed Bed System</li> <li>Lagoons and Ponds</li> </ul>
Level	6	7	8	9	10	11
Technology Units	<ul style="list-style-type: none"> <li>Empty Unit</li> <li>Secondary Settler</li> <li>Microfiltration</li> <li>Reverse Osmosis</li> <li>Chemical Precipitation</li> </ul>	<ul style="list-style-type: none"> <li>Empty Unit</li> <li>Physical Filtration</li> <li>Microfiltration</li> <li>Reverse Osmosis</li> <li>Chemical Precipitation</li> </ul>	<ul style="list-style-type: none"> <li>Empty Unit</li> <li>Physical Irradiation</li> <li>Ozonation</li> </ul>	<ul style="list-style-type: none"> <li>Empty Unit</li> <li>Air Stripping</li> <li>Ammonia Stripping</li> </ul>	<ul style="list-style-type: none"> <li>Empty Unit</li> <li>Chlorine Disinfection</li> <li>Chlorating Disinfection</li> </ul>	<ul style="list-style-type: none"> <li>Empty Unit</li> <li>GAC Adsorption</li> <li>Infiltration Basin</li> </ul>
SOLID LINE OF WTS						
Level	12	13	14	15	16	17
Technology Units	<ul style="list-style-type: none"> <li>Empty Unit</li> <li>Sludge Storage Tank</li> <li>Sludge Thickening Tank</li> </ul>	<ul style="list-style-type: none"> <li>Empty Unit</li> <li>Sludge Dewatering Bed</li> <li>Sludge C-G Drying</li> <li>Sludge V + A Stripping</li> <li>Sludge CWOP-UASB + A Stripping</li> <li>Sludge Hydrolysis + UASB</li> <li>Anaerobic Digestion</li> <li>Aerobic Digestion</li> <li>Aerobic-Anaerobic Digestion</li> </ul>	<ul style="list-style-type: none"> <li>Empty Unit</li> <li>Filter and Belt Permanent</li> <li>Thermal Process</li> <li>Thermo-Chemical Liquefaction</li> </ul>	<ul style="list-style-type: none"> <li>Empty Unit</li> <li>Sludge Dewatering Bed (II)</li> </ul>	<ul style="list-style-type: none"> <li>Empty Unit</li> <li>Physical Irradiation</li> </ul>	<ul style="list-style-type: none"> <li>Empty Unit</li> <li>Chemical Fixation</li> <li>Incineration</li> <li>Thermal Building Materials</li> </ul>

Table 2: Lower and upper limits (in mg/l) on the ten liquid state variables of the wastewater treatment system for the liquid line (Levels 1-11).

Entering ▼	Liq-COD	Liq-SS	Liq-orgN	Liq-ammN	Liq-nitN
Level 1	200 210	220 231	15 15.75	25 26.25	0 0.01
Level 2	52 210	59.4 231	4.5 15.75	2 26.25	0 0.01
Level 3	33.8 210	5.94 115.5	1.35 14.9625	1.8 28.875	0 0.01
Level 5	27.04 199.5	5.346 115.5	1.08 14.214375	1.08 23.1	1.314 42.1675
Level 6	0.5408 69.825	0.10692 7000	0.162 100	0 21.945	0.1314 122.26675
Level 7	0.05408 69.825	1.07(10 <sup>-3</sup> ) 350	0.0162 100	0 21.945	0.01314 122.26675
Level 8	5.408(10 <sup>-3</sup> ) 62.8425	1.07(10 <sup>-5</sup> ) 52.5	0.00162 70	0 21.945	1.314(10 <sup>-3</sup> ) 122.26675
Level 9	4.3264(10 <sup>-3</sup> ) 59.700375	9.6228(10 <sup>-6</sup> ) 52.5	1.296(10 <sup>-3</sup> ) 66.5	0 17.556	1.314(10 <sup>-3</sup> ) 170.3263
Level 10	8.6528(10 <sup>-4</sup> ) 47.7603	9.6228(10 <sup>-6</sup> ) 52.5	1.296(10 <sup>-3</sup> ) 66.5	0 14.9226	1.314(10 <sup>-3</sup> ) 170.3263
Level 11	8.6528(10 <sup>-4</sup> ) 47.7603	9.6228(10 <sup>-6</sup> ) 52.5	1.296(10 <sup>-3</sup> ) 66.5	0 14.9226	1.314(10 <sup>-3</sup> ) 170.3263
Entering ▼	Liq-totP	Liq-HM	Liq-SOCs	Liq-pathogens	Liq-viruses
Level 1	8 8.4	0.01 0.0105	15 15.75	5.00(10 <sup>7</sup> ) 5.25(10 <sup>7</sup> )	100 105
Level 2	2 8.4	0.0035 0.0105	3.75 15.75	1.75(10 <sup>7</sup> ) 5.25(10 <sup>7</sup> )	40 105
Level 3	0.2 7.98	0.000175 0.00945	0.375 14.175	1.75(10 <sup>6</sup> ) 3.15(10 <sup>7</sup> )	4 63
Level 5	0.2 7.98	0.000175 0.00945	0.05625 7.0875	0 3.15(10 <sup>6</sup> )	0.08 6.3
Level 6	0.01 10	1.75(10 <sup>-6</sup> ) 0.00567	5.625(10 <sup>-6</sup> ) 4.2525	0 1.89(10 <sup>6</sup> )	8(10 <sup>-6</sup> ) 3.78
Level 7	0 10	3.5(10 <sup>-8</sup> ) 0.00567	2.8125(10 <sup>-7</sup> ) 4.2525	0 1.89(10 <sup>6</sup> )	4(10 <sup>-8</sup> ) 3.78
Level 8	0 8	7(10 <sup>-10</sup> ) 0.001701	1.4063(10 <sup>-8</sup> ) 1.063125	0 283500	2(10 <sup>-10</sup> ) 0.567
Level 9	0 8	7(10 <sup>-10</sup> ) 0.001701	2.1094(10 <sup>-9</sup> ) 0.5315625	0 28350	4(10 <sup>-12</sup> ) 0.0567
Level 10	0 8	7(10 <sup>-10</sup> ) 0.001701	3.164(10 <sup>-10</sup> ) 0.26578125	0 4252.5	4(10 <sup>-14</sup> ) 0.008505
Level 11	0 8	7(10 <sup>-10</sup> ) 0.001701	3.164(10 <sup>-10</sup> ) 0.3189375	0 212.625	8(10 <sup>-15</sup> ) 0.0025515

Table 3: Lower and upper limits (in mg/l) on the ten sludge state variables of the wastewater treatment system for the liquid line (Levels 1-11).

Entering ▼	SI-Vol	SI-WC	SI-orgC	SI-inorgC	SI-orgN
Level 2	0.0575 0.453	0.0566 0.451	197.36 5328.72	56.4 1998.27	3.595 169.853
Level 3	1.0575 100.405	0.86 99.9	3248.395 9.23(10 <sup>6</sup> )	3167.25 9.41(10 <sup>6</sup> )	86.93 6.66(10 <sup>5</sup> )
Level 6	2.057 203.69	1.842 203.086	8048.395 1.03(10 <sup>7</sup> )	4367.25 9.826(10 <sup>6</sup> )	206.93 7.07(10 <sup>5</sup> )
Level 7	2.057 2728.69	1.842 2725.56	8048.47 2.4(10 <sup>7</sup> )	4367.266 1.89(10 <sup>7</sup> )	206.932 1.48(10 <sup>6</sup> )
Level 8	3.057 3078.69	2.802 3075.22	8348.47 2.455(10 <sup>7</sup> )	4667.266 1.953(10 <sup>7</sup> )	221.932 1.533(10 <sup>6</sup> )
Entering ▼	SI-ammN	SI-totP	SI-HM	SI-SOCs	SI-pathogens
Level 2	0.024 1.698	507.94 37705.5	0 0.83	152.381 14883.75	0 3.721(10 <sup>9</sup> )
Level 3	0.3545 3999.77	552.1 5.14(10 <sup>5</sup> )	0.126 629.252	341.62 9.08(10 <sup>5</sup> )	3.81(10 <sup>8</sup> ) 1.79(10 <sup>12</sup> )
Level 6	1.1545 4402.6	553.014 8.194(10 <sup>5</sup> )	0.1285 1121.44	342.3 1.3(10 <sup>6</sup> )	3.81(10 <sup>8</sup> ) 1.87(10 <sup>12</sup> )
Level 7	1.1545 59813.71	553.014 4.72(10 <sup>11</sup> )	0.1285 5.583(10 <sup>8</sup> )	342.3 2.932(10 <sup>11</sup> )	3.81(10 <sup>8</sup> ) 7.445(10 <sup>16</sup> )
Level 8	1.1545 67494.46	553.014 4.72(10 <sup>11</sup> )	0.1285 5.583(10 <sup>8</sup> )	342.3 2.93(10 <sup>11</sup> )	3.81(10 <sup>8</sup> ) 7.445(10 <sup>16</sup> )

Table 4: Lower and upper limits (in mg/l) on the ten sludge state variables of the wastewater treatment system for the solid line (Levels 12-17).

Entering ▼	SI-Vol	SI-WC	SI-orgC	SI-inorgC	SI-orgN
Level 12	3.0575 3078.691	0.001 0.9999	2.712 8.03(10 <sup>6</sup> )	1.516 6.387(10 <sup>6</sup> )	7.21(10 <sup>-2</sup> ) 5.0125(10 <sup>5</sup> )
Level 13	1 3078.691	0.001 0.996	2.441 8.03(10 <sup>6</sup> )	1.516 6.387(10 <sup>6</sup> )	6.5(10 <sup>-2</sup> ) 5.0125(10 <sup>5</sup> )
Level 14	1 3078.691	9.1(10 <sup>-5</sup> ) 0.999	0.122 8.03(10 <sup>6</sup> )	0.455 6.387(10 <sup>6</sup> )	3.24(10 <sup>-3</sup> ) 5.0125(10 <sup>5</sup> )
Level 15	1 3078.691	0.05 0.8	0.0122 8.03(10 <sup>6</sup> )	0.273 6.387(10 <sup>6</sup> )	4.866(10 <sup>-4</sup> ) 5.0125(10 <sup>5</sup> )
Level 16	1 3078.691	0.005 0.6	0.0122 8.03(10 <sup>6</sup> )	0.273 6.387(10 <sup>6</sup> )	4.866(10 <sup>-4</sup> ) 5.0125(10 <sup>5</sup> )
Level 17	1 3078.691	0.005 0.6	0.0122 8.03(10 <sup>6</sup> )	0.273 6.387(10 <sup>6</sup> )	4.866(10 <sup>-4</sup> ) 5.0125(10 <sup>5</sup> )
Entering ▼	SI-ammN	SI-totP	SI-HM	SI-SOCs	SI-pathogens
Level 12	3.75(10 <sup>-4</sup> ) 2.21(10 <sup>4</sup> )	0.1796 1.544(10 <sup>11</sup> )	4.173(10 <sup>-5</sup> ) 1.826(10 <sup>8</sup> )	0.1112 9.6(10 <sup>10</sup> )	1.24(10 <sup>3</sup> ) 2.435(10 <sup>16</sup> )
Level 13	3.75(10 <sup>-4</sup> ) 2.21(10 <sup>4</sup> )	0.1796 1.544(10 <sup>11</sup> )	4.173(10 <sup>-5</sup> ) 1.826(10 <sup>8</sup> )	0.1056 9.6(10 <sup>10</sup> )	6.187(10 <sup>4</sup> ) 2.435(10 <sup>16</sup> )
Level 14	3.75(10 <sup>-5</sup> ) 2.21(10 <sup>4</sup> )	0.028 1.544(10 <sup>11</sup> )	6.506(10 <sup>-6</sup> ) 1.826(10 <sup>8</sup> )	0.0317 9.6(10 <sup>10</sup> )	61.87 2.435(10 <sup>16</sup> )
Level 15	3.75(10 <sup>-6</sup> ) 2.21(10 <sup>4</sup> )	0.0168 1.544(10 <sup>11</sup> )	6.506(10 <sup>-6</sup> ) 1.826(10 <sup>8</sup> )	0 9.6(10 <sup>10</sup> )	18.561 2.435(10 <sup>16</sup> )
Level 16	3.56(10 <sup>-6</sup> ) 2.21(10 <sup>4</sup> )	0.0168 1.544(10 <sup>11</sup> )	6.506(10 <sup>-6</sup> ) 1.826(10 <sup>8</sup> )	0 9.6(10 <sup>10</sup> )	0.18561 2.435(10 <sup>15</sup> )
Level 17	3.56(10 <sup>-6</sup> ) 2.21(10 <sup>4</sup> )	0.0168 1.544(10 <sup>11</sup> )	6.506(10 <sup>-6</sup> ) 1.826(10 <sup>8</sup> )	0 9.6(10 <sup>10</sup> )	1.856(10 <sup>-4</sup> ) 2.435(10 <sup>13</sup> )

Table 5: Minimum and maximum values (in mg/l) of the ten liquid state variables for the wastewater exiting the Liquid Line. Targets are approximately 10% of the maximums. For the quintic penalty function, differences  $\Delta$  between the knots are calculated as  $0.5 \times (\text{maximum} - \text{target})$ . Penalty coefficients are calculated as  $2000 / (\text{maximum} - \text{target})$ .

<b>Effluent</b> ↓	<b>L-COD</b>	<b>L-SS</b>	<b>L-orgN</b>	<b>L-ammN</b>	<b>L-nitN</b>
Minimum	8.6528( $10^{-4}$ )	9.6228( $10^{-6}$ )	1.296( $10^{-3}$ )	0	1.314( $10^{-3}$ )
Maximum	47.7603	52.5	66.5	14.9226	170.3263
Target	5	5.5	7	1.5	16
$\Delta$	21.4	24	30	6.7	77.2
Penalty	47	43	34.	149	13

<b>Effluent</b> ↓	<b>L-totP</b>	<b>L-HM</b>	<b>L-SOCs</b>	<b>L-pathogens</b>	<b>L-viruses</b>
Minimum	0	7( $10^{-10}$ )	3.164( $10^{-10}$ )	0	8( $10^{-15}$ )
Maximum	8	0.001701	0.32	212.625	0.0025515
Target	0.8	0.00015	0.04	20	0.0003
$\Delta$	3.6	0.00078	0.1395	96.313	0.00112
Penalty	278	1.29( $10^6$ )	7.2( $10^3$ )	10.4	8.88( $10^5$ )

Table 6: Minimum and maximum values (in mg/l) of the ten solid state variables for the solid exiting the Solid Line. Targets are approximately 10% of the maximums. For the quintic penalty function, differences  $\Delta$  between the knots are calculated as  $0.5 \times (\text{maximum} - \text{target})$ . Penalty coefficients are calculated as  $2000 / (\text{maximum} - \text{target})$ .

<b>Effluent</b> ↓	<b>S-Vol</b>	<b>S-WC</b>	<b>S-orgC</b>	<b>S-inorgC</b>	<b>S-orgN</b>
Minimum	1	0	0.0122	0.082	4.87( $10^{-4}$ )
Maximum	3078.691	0.30	8.03( $10^6$ )	8.942( $10^6$ )	5.0125( $10^5$ )
Target	306	0.03	6.08( $10^5$ )	6.83( $10^5$ )	3.81( $10^4$ )
$\Delta$	1386	0.135	3.71( $10^6$ )	4.13( $10^6$ )	2.316( $10^5$ )
Penalty	0.72	7407.41	2.7( $10^{-4}$ )	2.4( $10^{-4}$ )	4.32( $10^{-3}$ )

<b>Effluent</b> ↓	<b>S-ammN</b>	<b>S-totP</b>	<b>S-HM</b>	<b>S-SOCs</b>	<b>S-pathogens</b>
Minimum	1.425( $10^{-6}$ )	0.0168	6.5( $10^{-6}$ )	0	1.856( $10^{-6}$ )
Maximum	2.21( $10^4$ )	1.5442( $10^{11}$ )	1.826( $10^8$ )	9.588( $10^{10}$ )	2.435( $10^{13}$ )
Target	1685	1.472( $10^5$ )	453	2.74( $10^5$ )	1.38( $10^8$ )
$\Delta$	1.02( $10^4$ )	7.721( $10^{10}$ )	913( $10^7$ )	4.794( $10^{10}$ )	1.22( $10^{13}$ )
Penalty	0.0981	1.3( $10^{-8}$ )	1.1( $10^{-5}$ )	2.09( $10^{-8}$ )	8.21( $10^{-11}$ )



Table 9: The interlevel importance questions for six objectives.

	Which one of the following pairs of objectives is more important in terms of improvement from the worst value to the best value?			Given the more important objective, how many times is this objective more important than the other? (Note: "E qual" = 1)									
EC vs. EC				1	2	3	4	5	6	7	8	9	
L1 vs. L2	<input checked="" type="checkbox"/> $EC_1$	<input type="checkbox"/> $EC_2$	<input type="checkbox"/> Equal	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
L2 vs. L3	<input checked="" type="checkbox"/> $EC_2$	<input type="checkbox"/> $EC_3$	<input type="checkbox"/> Equal	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
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L16 vs. L17	<input type="checkbox"/> $EC_{16}$	<input checked="" type="checkbox"/> $EC_{17}$	<input type="checkbox"/> Equal	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
S vs. S													
L1 vs. L2	<input checked="" type="checkbox"/> $S_1$	<input type="checkbox"/> $S_2$	<input type="checkbox"/> Equal	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
L2 vs. L3	<input type="checkbox"/> $S_2$	<input type="checkbox"/> $S_3$	<input checked="" type="checkbox"/> Equal	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
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L16 vs. L17	<input type="checkbox"/> $S_{16}$	<input checked="" type="checkbox"/> $S_{17}$	<input type="checkbox"/> Equal	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
NR vs. NR													
L1 vs. L2	<input type="checkbox"/> $NR_1$	<input checked="" type="checkbox"/> $NR_2$	<input type="checkbox"/> Equal	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
L2 vs. L3	<input checked="" type="checkbox"/> $NR_2$	<input type="checkbox"/> $NR_3$	<input type="checkbox"/> Equal	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
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L16 vs. L17	<input type="checkbox"/> $NR_{16}$	<input checked="" type="checkbox"/> $NR_{17}$	<input type="checkbox"/> Equal	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
O vs. O													
L1 vs. L2	<input checked="" type="checkbox"/> $O_1$	<input type="checkbox"/> $O_2$	<input type="checkbox"/> Equal	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
L2 vs. L3	<input type="checkbox"/> $O_2$	<input type="checkbox"/> $O_3$	<input checked="" type="checkbox"/> Equal	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
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L16 vs. L17	<input type="checkbox"/> $O_{16}$	<input checked="" type="checkbox"/> $O_{17}$	<input type="checkbox"/> Equal	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
R vs. R													
L1 vs. L2	<input checked="" type="checkbox"/> $R_1$	<input type="checkbox"/> $R_2$	<input type="checkbox"/> Equal	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
L2 vs. L3	<input checked="" type="checkbox"/> $R_2$	<input type="checkbox"/> $R_3$	<input type="checkbox"/> Equal	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
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L16 vs. L17	<input type="checkbox"/> $R_{16}$	<input checked="" type="checkbox"/> $R_{17}$	<input type="checkbox"/> Equal	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
GD vs. GD													
L1 vs. L2	<input checked="" type="checkbox"/> $GD_1$	<input type="checkbox"/> $GD_2$	<input type="checkbox"/> Equal	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
L2 vs. L3	<input type="checkbox"/> $GD_2$	<input checked="" type="checkbox"/> $GD_3$	<input type="checkbox"/> Equal	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
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L16 vs. L17	<input type="checkbox"/> $GD_{16}$	<input checked="" type="checkbox"/> $GD_{17}$	<input type="checkbox"/> Equal	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Table 10: Weight vectors for GM method

$\mathbf{W}_1$	(0.154237, 0.058427, 0.421142, 0.256685, 0.036603, 0.072906)
$\mathbf{W}_2$	(0.218300, 0.117765, 0.273487, 0.195936, 0.139935, 0.054577)
$\mathbf{W}_3$	(0.244031, 0.106403, 0.214016, 0.119192, 0.203495, 0.112864)
$\mathbf{W}_4$	(0.244031, 0.106403, 0.214016, 0.119192, 0.203495, 0.112864)
$\mathbf{W}_5$	(0.244031, 0.106403, 0.214016, 0.119192, 0.203495, 0.112864)
$\mathbf{W}_6$	(0.231888, 0.137621, 0.225310, 0.101896, 0.219167, 0.084118)
$\mathbf{W}_7$	(0.231888, 0.137621, 0.225310, 0.101896, 0.219167, 0.084118)
$\mathbf{W}_8$	(0.221956, 0.142302, 0.204020, 0.089873, 0.225492, 0.116357)
$\mathbf{W}_9$	(0.211932, 0.147220, 0.196833, 0.104796, 0.214795, 0.124425)
$\mathbf{W}_{10}$	(0.193660, 0.147616, 0.188594, 0.128934, 0.196043, 0.145154)
$\mathbf{W}_{11}$	(0.191603, 0.154427, 0.187325, 0.131833, 0.189292, 0.145520)
$\mathbf{W}_{12}$	(0.186320, 0.161504, 0.186168, 0.132277, 0.174369, 0.159363)
$\mathbf{W}_{13}$	(0.183047, 0.161063, 0.179718, 0.138594, 0.167228, 0.170350)
$\mathbf{W}_{14}$	(0.186209, 0.166115, 0.183183, 0.124514, 0.171779, 0.168199)
$\mathbf{W}_{15}$	(0.176766, 0.160269, 0.174110, 0.146163, 0.163228, 0.179465)
$\mathbf{W}_{16}$	(0.180087, 0.164718, 0.177622, 0.127089, 0.167485, 0.182998)
$\mathbf{W}_{17}$	(0.178291, 0.167024, 0.177371, 0.135758, 0.164072, 0.177484)

Table 11: Weight vectors for SGM method

$\mathbf{W}_1$	(0.154237, 0.058427, 0.421142, 0.256685, 0.036603, 0.072906)
$\mathbf{W}_2$	(0.218300, 0.117765, 0.273487, 0.195936, 0.139935, 0.054577)
$\mathbf{W}_3$	(0.249305, 0.087228, 0.150558, 0.076308, 0.253163, 0.183438)
$\mathbf{W}_4$	(0.249305, 0.087228, 0.150558, 0.076308, 0.253163, 0.183438)
$\mathbf{W}_5$	(0.249305, 0.087228, 0.150558, 0.076308, 0.253163, 0.183438)
$\mathbf{W}_6$	(0.223666, 0.164886, 0.212225, 0.063104, 0.266724, 0.069395)
$\mathbf{W}_7$	(0.223666, 0.164886, 0.212225, 0.063104, 0.266724, 0.069395)
$\mathbf{W}_8$	(0.192707, 0.152082, 0.165406, 0.060352, 0.242291, 0.187161)
$\mathbf{W}_9$	(0.174404, 0.160278, 0.165265, 0.138625, 0.189626, 0.171803)
$\mathbf{W}_{10}$	(0.097431, 0.144250, 0.145907, 0.243649, 0.102808, 0.265956)
$\mathbf{W}_{11}$	(0.149496, 0.181108, 0.171078, 0.177404, 0.138297, 0.18261)
$\mathbf{W}_{12}$	(0.134220, 0.185263, 0.156601, 0.131842, 0.097592, 0.294482)
$\mathbf{W}_{13}$	(0.132221, 0.149261, 0.126301, 0.152176, 0.097900, 0.342142)
$\mathbf{W}_{14}$	(0.183986, 0.191534, 0.181199, 0.076517, 0.166317, 0.200446)
$\mathbf{W}_{15}$	(0.107444, 0.114419, 0.106874, 0.274970, 0.099282, 0.297011)
$\mathbf{W}_{16}$	(0.173007, 0.176721, 0.172702, 0.066514, 0.168428, 0.242628)
$\mathbf{W}_{17}$	(0.163754, 0.188783, 0.173912, 0.171678, 0.141776, 0.160098)

Table 12: Consistency indices (CI) for GM and SGM methods

	<b>GM Method</b>	<b>SGM Method</b>
CI <sub>1</sub>	0.054527	0.054527
CI <sub>2</sub>	0.005730	0.005730
CI <sub>3</sub>	0.003273	0.012704
CI <sub>4</sub>	0.003273	0.012704
CI <sub>5</sub>	0.003273	0.012704
CI <sub>6</sub>	0.002739	0.010679
CI <sub>7</sub>	0.002739	0.010679
CI <sub>8</sub>	0.001420	0.001159
CI <sub>9</sub>	0.001117	0.000310
CI <sub>10</sub>	0.000546	0.002617
CI <sub>11</sub>	0.000380	0.000291
CI <sub>12</sub>	0.000247	0.001748
CI <sub>13</sub>	0.000196	0.001932
CI <sub>14</sub>	0.000148	0.000214
CI <sub>15</sub>	0.000140	0.004674
CI <sub>16</sub>	0.000109	0.000517
CI <sub>17</sub>	0.000091	0.000057

Table 13: Selected technologies using GM method with 2209 design points.

L level	Technology Unit	Count	L level	Technology Unit	Count
1	Flow Equalization Tank	0	10	Empty Unit	2205
	<i>Yellow Water Separation</i>	2209		Chlorine Disinfection	4
	<i>Yellow &amp; Black Water Separation</i>	0		Chlorating Disinfection	0
			11	Empty Unit	0
2	Empty Unit	0		GAC Adsorption	2209
	<i>Vortex SSO</i>	1471		Infiltration Basin	0
	Sedimentation Tank	0	12	Empty Unit	0
	Chemical Precipitation	738		Sludge Storage Tank	6
3	Empty Unit	0		Sludge Thickening Tank	2203
	Physical Irradiation	8	13	Empty Unit	0
	Ozonation	2201		Sludge Dewatering Bed	1461
4	Empty Unit	2209		Sludge C-G Drying	0
				Sludge V + A Stripping	0
5	Empty Unit	1		CWOP-UASB + A	0
	Activated Sludge (C)	0		Stripping	0
	Activated Sludge (C, N)	10		Sludge Hydrolysis + UASB	0
	Activated Sludge (C, P)	0		Anaerobic Digestion	0
	Activated Sludge (C, P, N)	0		Aerobic Digestion	0
	<i>High Biomass Act. Sludge (C, N)</i>	0		Aerobic-Anaerobic Digestion	748
	Activated Sludge (N)	11	14	Empty Unit	688
	<i>Multi-reactor/Deep</i>	0		Filter and Belt	981
	A-B System	1		Permanent Thermal Process	332
	Trickling Filter	65		Thermo-Chemical	208
	Rotating Biological Contractors	0		Liquefaction	
	UASB System	0	15	Empty Unit	712
	Reed Bed System	0		Sludge Dewatering Bed (II)	1497
	Lagoons and Ponds	0	16	Empty Unit	28
	<i>UASB+Activated Sludge (C, P, N)</i>	2121		Physical Irradiation	2181
6	Empty Unit	0	17	Empty Unit	980
	Secondary Settler	26		Chemical Fixation	1002
	Microfiltration	50		Incineration	33
	Reverse Osmosis	13		Thermal Building Materials	194
	Chemical Precipitation	2120			
7	Empty Unit	0			
	Physical Filtration	220			
	Microfiltration	178			
	Reverse Osmosis	1677			
	Chemical Precipitation	134			
8	Empty Unit	2			
	<i>Physical Irradiation</i>	1356			
	Ozonation	851			
9	Empty Unit	436			
	Air Stripping	1739			
	Ammonia Stripping	34			

Table 14: Selected technologies using SGM method with 2209 design points.

L level	Technology Unit	Count
1	Flow Equalization Tank	0
	<i>Yellow Water Separation</i>	2209
	<i>Yellow &amp; Black Water Separation</i>	0
2	Empty Unit	0
	<i>Vortex SSO</i>	1914
	Sedimentation Tank	0
	Chemical Precipitation	295
3	Empty Unit	0
	Physical Irradiation	5
	Ozonation	2204
4	Empty Unit	2209
5	Empty Unit	1
	Activated Sludge (C)	0
	Activated Sludge (C, N)	308
	Activated Sludge (C, P)	0
	Activated Sludge (C, P, N)	5
	<i>High Biomass Act. Sludge (C, N)</i>	0
	Activated Sludge (N)	135
	<i>Multi-reactor/Deep</i>	0
	A-B System	0
	Trickling Filter	52
	Rotating Biological Contractors	0
	UASB System	0
	Reed Bed System	0
	Lagoons and Ponds	0
	<i>UASB+Activated Sludge (C, P, N)</i>	1708
6	Empty Unit	0
	Secondary Settler	23
	Microfiltration	63
	Reverse Osmosis	9
	Chemical Precipitation	2114
7	Empty Unit	0
	Physical Filtration	229
	Microfiltration	190
	Reverse Osmosis	1655
	Chemical Precipitation	135
8	Empty Unit	2
	<i>Physical Irradiation</i>	1349
	Ozonation	858
9	Empty Unit	436
	Air Stripping	1739
	Ammonia Stripping	34
10	Empty Unit	2205
	Chlorine Disinfection	4
	Chlorating Disinfection	0
11	Empty Unit	0
	GAC Adsorption	2209
	Infiltration Basin	0

L level	Technology Unit	Count
12	Empty Unit	0
	Sludge Storage Tank	6
	Sludge Thickening Tank	2203
13	Empty Unit	0
	Sludge Dewatering Bed	1411
	Sludge C-G Drying	0
	Sludge V + A Stripping	0
	CWOP-UASB + A Stripping	0
	Sludge Hydrolysis + UASB	0
	Anaerobic Digestion	0
	Aerobic Digestion	0
	Aerobic-Anaerobic Digestion	798
14	Empty Unit	690
	Filter and Belt	982
	Permanent Thermal Process	326
	Thermo-Chemical Liquefaction	211
15	Empty Unit	734
	Sludge Dewatering Bed (II)	1475
16	Empty Unit	26
	Physical Irradiation	2183
17	Empty Unit	1032
	Chemical Fixation	929
	Incineration	33
	Thermal Building Materials	215

Table 15: Selected technologies using GM method with 12167 design points, where the fractions in square brackets denote the  $R^2$  values.

L level	Technology Unit	Count
1 [0.91]	Flow Equalization Tank	0
	<i>Yellow Water Separation</i>	12167
	<i>Yellow &amp; Black Water Separation</i>	0
2 [0.984]	Empty Unit	96
	<i>Vortex SSO</i>	12007
	Sedimentation Tank	3
	Chemical Precipitation	61
3 [0.975]	Empty Unit	0
	Physical Irradiation	0
	Ozonation	12167
4	Empty Unit	12167
5 [0.983]	Empty Unit	0
	Activated Sludge (C)	0
	Activated Sludge (C, N)	0
	Activated Sludge (C, P)	0
	Activated Sludge (C, P, N)	0
	<i>High Biomass Act. Sludge (C, N)</i>	0
	Activated Sludge (N)	0
	<i>Multi-reactor/Deep</i>	6016
	A-B System	0
	Trickling Filter	20
	Rotating Biological Contractors	0
	UASB System	113
	Reed Bed System	0
	Lagoons and Ponds	0
	<i>UASB+Activated Sludge (C, P, N)</i>	6018
6 [0.94]	Empty Unit	6
	Secondary Settler	3216
	Microfiltration	7547
	Reverse Osmosis	88
	Chemical Precipitation	1310
7 [0.974]	Empty Unit	1
	Physical Filtration	45
	Microfiltration	4678
	Reverse Osmosis	6647
	Chemical Precipitation	796
8 [0.94]	Empty Unit	286
	<i>Physical Irradiation</i>	516
	Ozonation	11365
9 [0.77]	Empty Unit	4125
	Air Stripping	8037
	Ammonia Stripping	5

L level	Technology Unit	Count
10 [0.832]	Empty Unit	12162
	Chlorine Disinfection	5
	Chlorating Disinfection	0
11 [0.999]	Empty Unit	0
	GAC Adsorption	12167
	Infiltration Basin	0
12 [0.999]	Empty Unit	0
	Sludge Storage Tank	44
	Sludge Thickening Tank	12123
13 [0.996]	Empty Unit	0
	Sludge Dewatering Bed	8094
	Sludge C-G Drying	0
	Sludge V + A Stripping	0
	CWOP-UASB + A Stripping	0
	Sludge Hydrolysis + UASB	0
	Anaerobic Digestion	0
	Aerobic Digestion	0
	Aerobic-Anaerobic Digestion	4073
14 [0.972]	Empty Unit	3903
	Filter and Belt	5379
	Permanent Thermal Process	1780
	Thermo-Chemical Liquefaction	1105
15 [0.995]	Empty Unit	3900
	Sludge Dewatering Bed (II)	8267
16 [0.921]	Empty Unit	152
	Physical Irradiation	12015
17 [0.988]	Empty Unit	5415
	Chemical Fixation	5493
	Incineration	212
	Thermal Building Materials	1047

Table 16: Selected technologies using SGM method with 12167 design points, where the fractions in square brackets denote the  $R^2$  values.

L level	Technology Unit	Count
1 [0.999]	Flow Equalization Tank <i>Yellow Water Separation</i> <i>Yellow &amp; Black Water Separation</i>	0 12167 0
2 [0.999]	Empty Unit <i>Vortex SSO</i> Sedimentation Tank Chemical Precipitation	0 12167 0 0
3 [0.431]	Empty Unit Physical Irradiation Ozonation	0 0 12167
4	Empty Unit	12167
5 [0.989]	Empty Unit Activated Sludge (C) Activated Sludge (C, N) Activated Sludge (C, P) Activated Sludge (C, P, N) <i>High Biomass Act. Sludge (C, N)</i> Activated Sludge (N) <i>Multi-reactor/Deep</i> A-B System Trickling Filter Rotating Biological Contractors UASB System Reed Bed System Lagoons and Ponds <i>UASB+Activated Sludge (C, P, N)</i>	0 0 0 0 0 0 0 0 0 55 0 0 0 0 0 12112
6 [0.84]	Empty Unit Secondary Settler Microfiltration Reverse Osmosis Chemical Precipitation	2 3504 7617 10 1034
7 [0.971]	Empty Unit Physical Filtration Microfiltration Reverse Osmosis Chemical Precipitation	1 25 4469 6980 692
8 [0.935]	Empty Unit <i>Physical Irradiation</i> Ozonation	283 522 11362
9 [0.77]	Empty Unit Air Stripping Ammonia Stripping	4121 8041 5

L level	Technology Unit	Count
10 [0.832]	Empty Unit Chlorine Disinfection Chlorating Disinfection	12162 5 0
11 [0.999]	Empty Unit GAC Adsorption Infiltration Basin	0 12167 0
12 [0.999]	Empty Unit Sludge Storage Tank Sludge Thickening Tank	0 18 12149
13 [0.996]	Empty Unit Sludge Dewatering Bed Sludge C-G Drying Sludge V + A Stripping CWOP-UASB + A Stripping Sludge Hydrolysis + UASB Anaerobic Digestion Aerobic Digestion Aerobic-Anaerobic Digestion	0 7809 0 0 0 0 0 0 0 4358
14 [0.972]	Empty Unit Filter and Belt Permanent Thermal Process Thermo-Chemical Liquefaction	3922 5396 1730 1119
15 [0.99]	Empty Unit Sludge Dewatering Bed (II)	3993 8174
16 [0.91]	Empty Unit Physical Irradiation	148 12019
17 [0.985]	Empty Unit Chemical Fixation Incineration Thermal Building Materials	5648 5127 219 1173

Figure 1: Levels and treatment technologies for the liquid line of the wastewater treatment system.

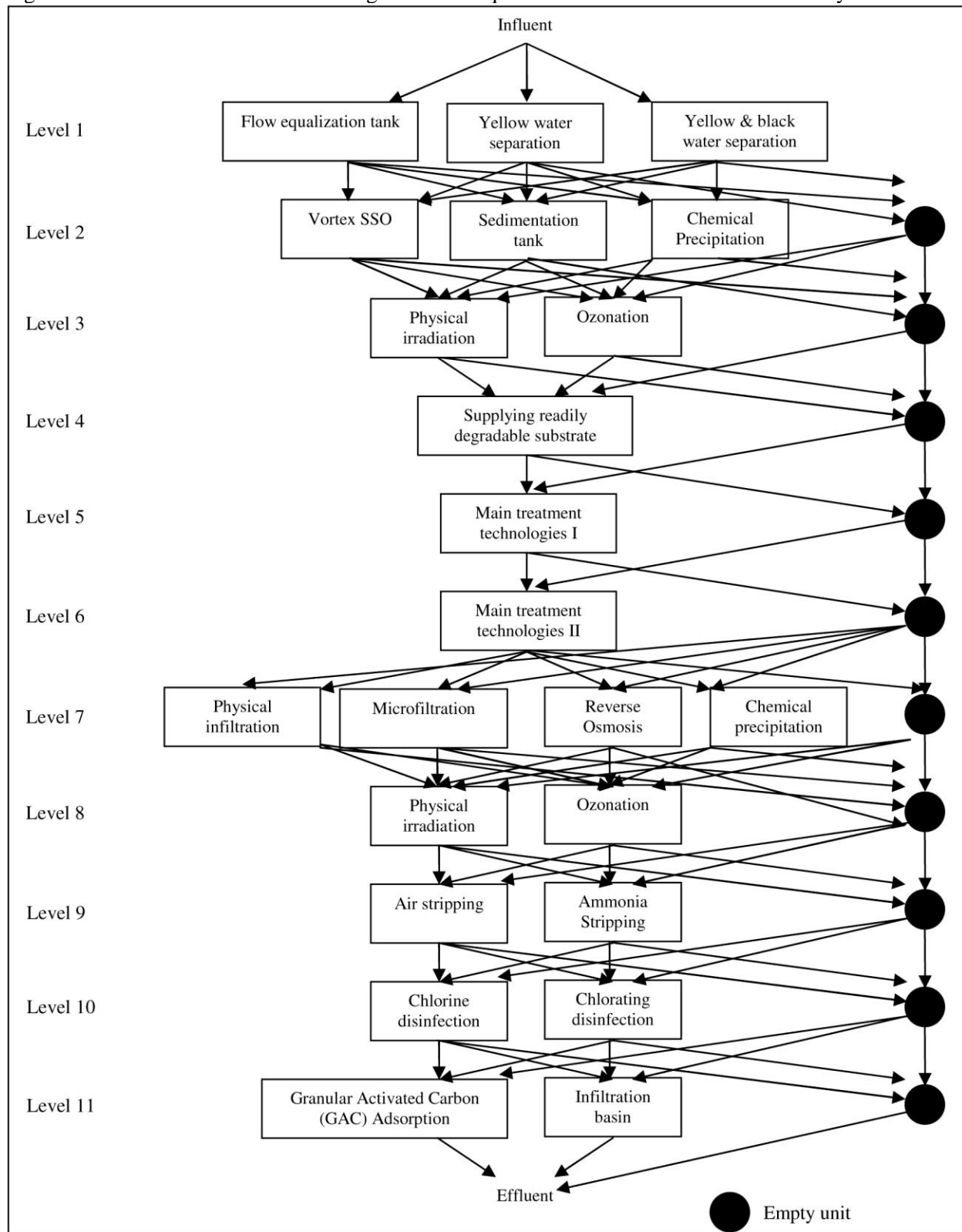


Figure 2: Levels and treatment technologies for the solid line of the wastewater treatment system.

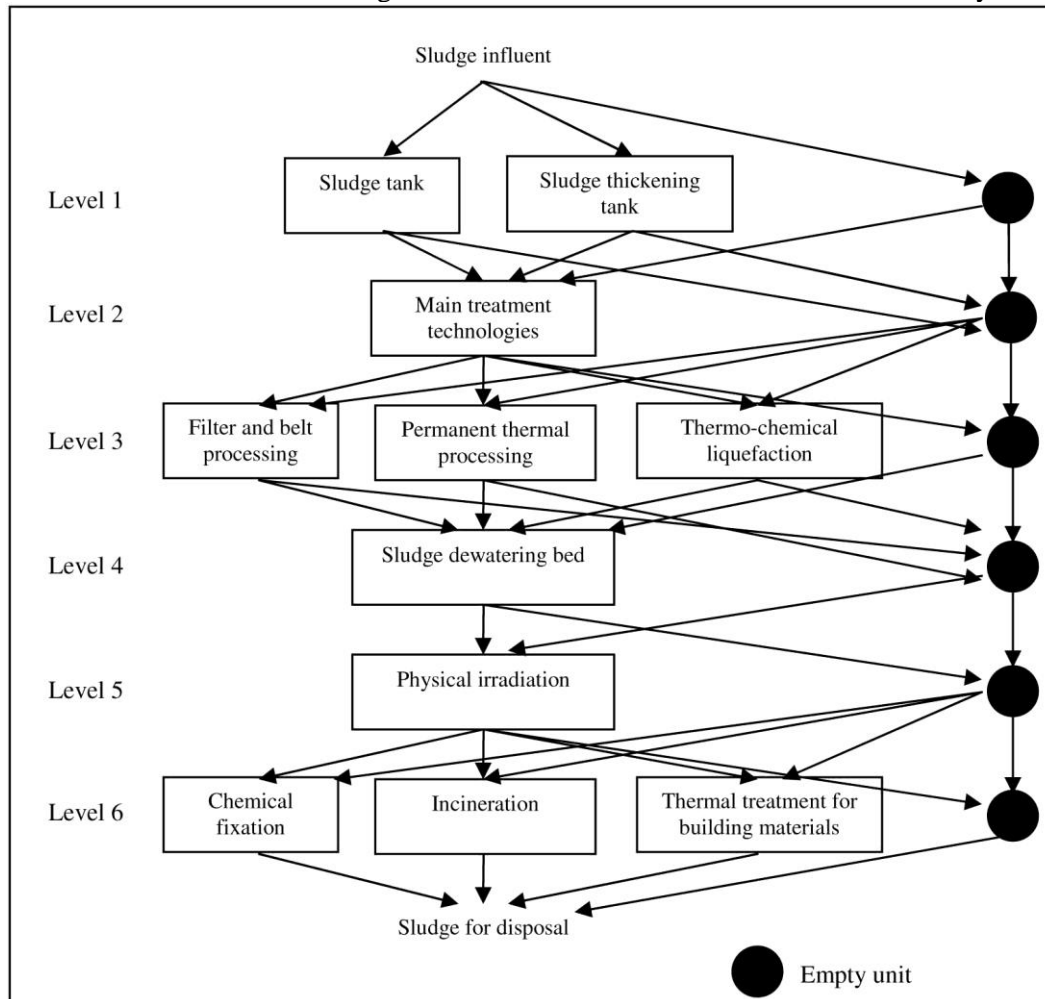


Figure 3: Basic formulation of multiple stage multiple objective optimization problem.

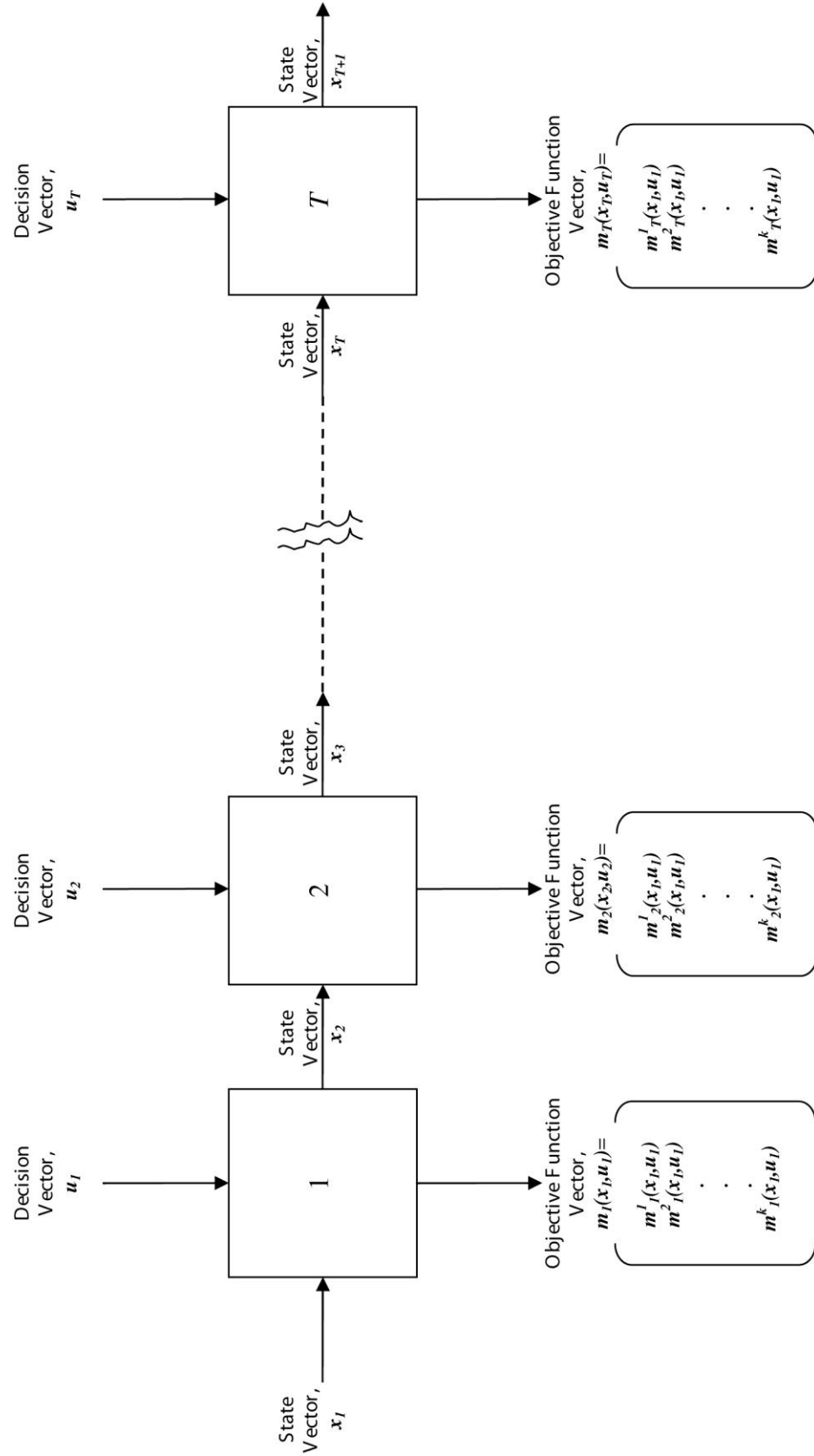


Figure 4: Scalarization using weighted-sum of objective functions approach.

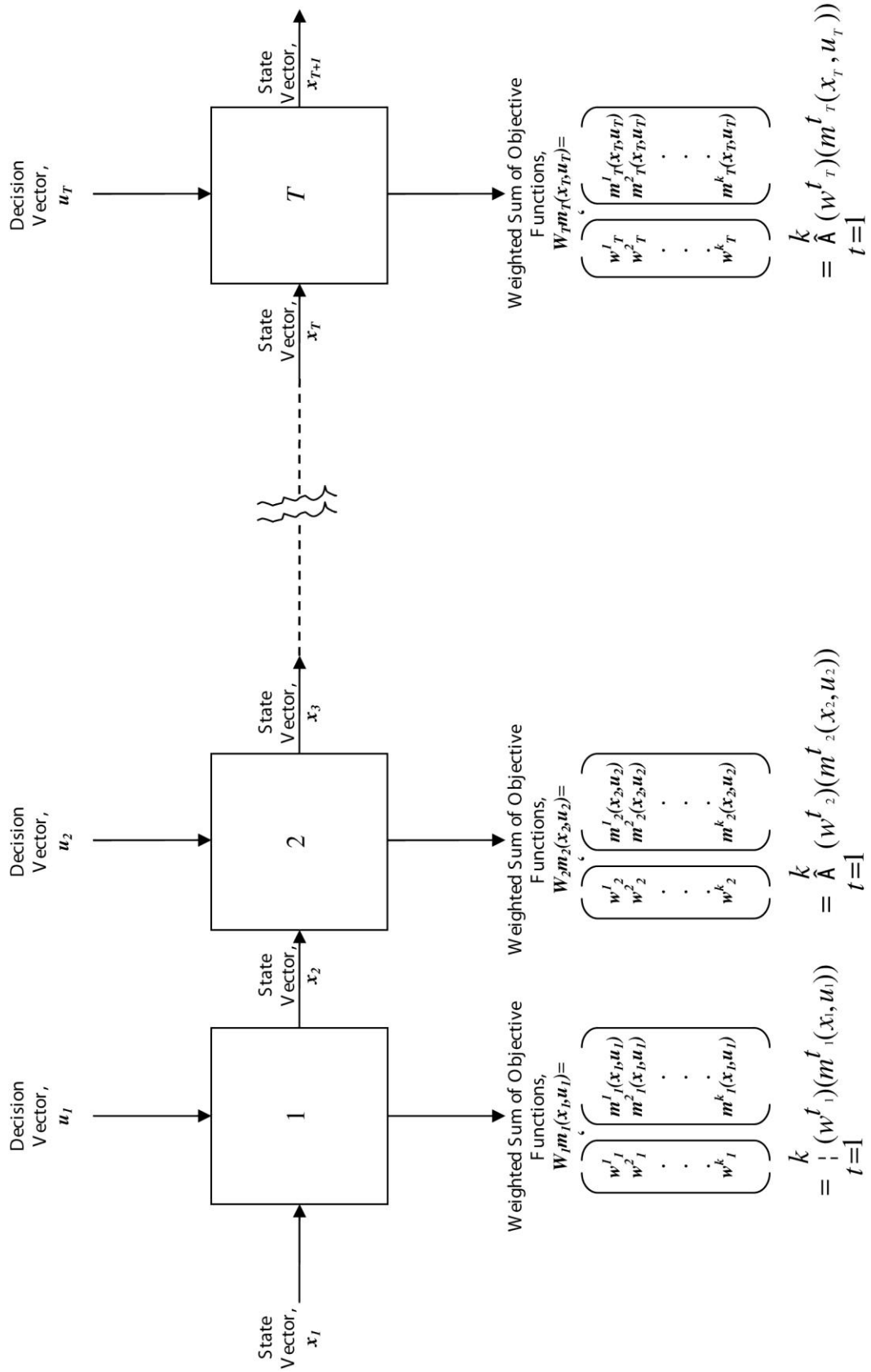


Figure 5: A typical multistage multiobjective model highlighting three phases in our methodology.

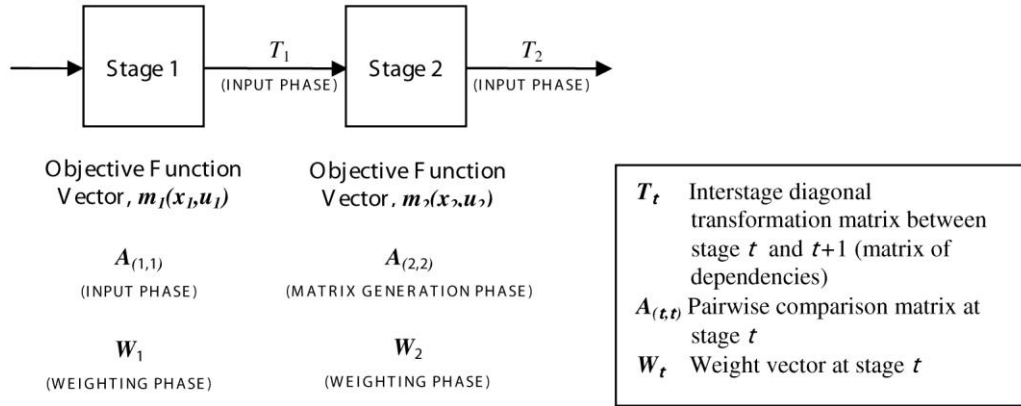


Figure 6: Consistency indices of pairwise comparison matrices over WTS levels.

