Study of Time Dependent Queuing Models of the National Airspace System

Na Wang[1], Chatabush Roongnat[2], Jay M. Rosenberger[1], P. K. Menon[4], Kamesh Subbarao[3], Prasenjit Sengupta[4] and Monish D. Tandale[4]

[1]Department of Industrial Engineering, The University of Texas at Arlington Box 19017, Arlington, TX 76019, U. S. A; [2]TransSolutions, 14600 Trinity Blvd, Fort Worth, TX 76155, U. S. A; [3]Department of Mechanical & Aerospace Engineering, The University of Texas at Arlington Box 19023, Arlington, TX 76019, U. S. A; [4]Optimal Synthesis Inc, 95 First Street, Suite 240, Los Altos, CA 94022, U. S. A.

Abstract

Queuing models provide an attractive and highly-efficient alternative to simulation for quantifying traffic flow efficiency. Stationary Markovian queuing models in which both inter-arrival times and service times are exponentially distributed have been studied by the National Airspace System (NAS). However, stationary queues cannot account for peaks and valleys in demand that are commonly observed in the National Airspace. Thus time dependent Markovian queuing models, which aim to capture the variation in demand during a day, have been studied. Furthermore, statistical analysis of real traffic data reveals that inter-arrival times and service times do not follow exponential distributions. As a subclass of phase-type distribution, Coxian distribution with the advantage of closely approximating any distribution without violating the Markov property has gained special importance on research in queuing systems. In this research, time-dependent Coxian queuing models $C_{m(t)}(t)/C_k/s/s$ for modeling the en route flight phase are developed as well, which are approximated by a piece-wise constant Coxian inter-arrival time distribution and a time-invariant Coxian service time distribution. Both the arrival rate and service rate are calibrated from data extracted from high-fidelity simulation runs driven by actual flying data. The number of aircraft in the system is regarded as a measure of the accuracy of queuing performance. Comparison results between time-dependent Markovian and Coxian queuing models are given in this paper. This study shows that time-dependent Markovian queues could capture the variation in demand as well as Coxian queues, with the advantage of mathematical and computational tractability.

Keyword: *National Airspace* *Time-dependent queuing models* *Coxian distribution* *Markovian queue* *queuing performance*

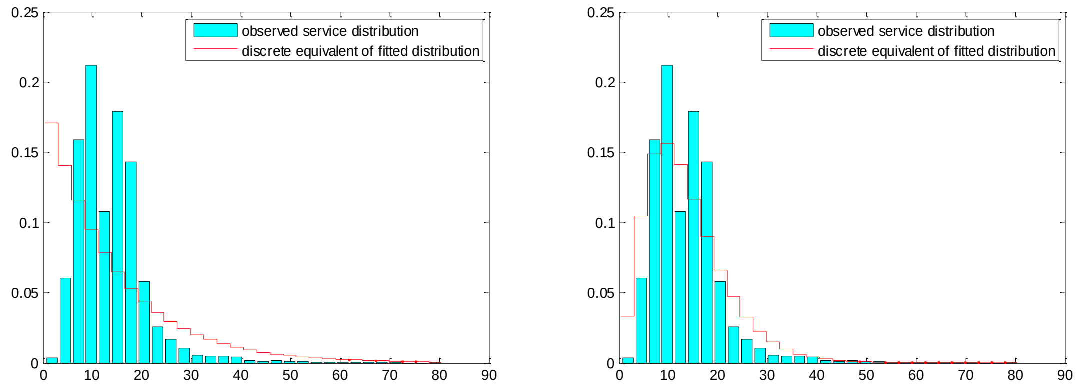
1 Introduction

Air transportation in the US system has dramatically changed in the past few decades. The National Airspace System (NAS) has increasingly become congested. The amount of traffic demand on the NAS frequently exceeds its available capacity, due to a large number of factors including bad weather, over-scheduling by the airlines, national security, and air traffic control equipment outages [1]. For the period of January through June 2000, delays increased by 13.6 percent from the same time period in 1999 [2]. In June alone, delays increased 20 percent. Delay results in huge economic loss to both passengers and industries. According to the U.S. Department of Transportation, in 1998, airline delays in the U.S. cost industries and passengers $4.5 billion [3]. Therefore, an efficient and effective air traffic management system is vital to the U.S. transportation infrastructure.

Air-traffic flow simulation software such as the Future ATM Concepts Evaluation Tool (FACET) can be used to quantify traffic flow efficiency. By using actual air traffic data from the Federal Aviation Administration (FAA), FACET analyzes the flight plan route and predicts trajectories for the climb, cruise, and descent phases of flight for each aircraft type, which can be further used to analyze congestion patterns of different sectors of the airspace by propagating the trajectories of proposed flights forward in time [2]. However, the time consuming and non-analytic characteristics of FACET are not amenable for conducting rapid trade studies. By contrast, modeling the air traffic flow by queuing models could provide quantitative information about the effects of the tools on operations of the NAS to facilitate tradeoff studies in an effective and time-efficient manner [4].

Queuing theory is first known from the work of A. K. Erlang of the Copenhagen Telephone Company in 1900s. Nowadays, it is used widely to analyze computer systems [5–7], communication systems [8–10] and transportation systems [11–15]. Yet, most research about queuing models focuses on non-stationary queuing models. However, in most real application, the arrival rate is non-stationary. For non-stationary, even if for moderately non-stationary Markovian queuing systems, research has shown that results of stationary models are quite inaccurate [16].

In this study, arrival rate (the amount of flying demand) is fluctuating widely during a day, such as increases during rush hour. In order to capture the variation in the arrival process, time dependent Markovian queuing models $M(t)/M/s/s$ are developed. Further statistical analysis shows that arrival and service times do not follow exponential distributions exactly. Since Coxian distributions have the advantage of approximating any arbitrary distribution without violating the Markov property, the Coxian distribution is employed to fit both inter-arrival and service times in this research. Results of fitting both the exponential distribution and a 3-phase Coxian distribution to data of service times are shown in figure 1a and 1b separately. Clearly the Coxian distribution fits the data better. Thus, for exploring more accurate queuing performance measures, time dependent Coxian queuing models $C_{m(t)}(t)/C_k/s/s$ are also developed.



Exponential distribution 3-phase Coxian distribution

Figure 1. Service Time Distributions

The remainder of Section 1, an overview of flight profiles and time dependent queuing models is introduced. Section 2 describes the models used in this research, overviews the Coxian distribution and solutions methods for $C_m/C_k/s/s$ models, and presents the methodology of solving $C_{m(t)}(t)/C_k/s/s$ queues. In Section 3, both $M(t)/M/s/s$ and $C_{m(t)}(t)/C_k/s/s$ queuing models are validated by data extracted from FACET. In addition, comparison between steady state approximation and expected transient state approximation results are shown. Conclusions and discussions are given in Section 4. Section 5 is acknowledgements.

1.1 Overview of Flight Profiles

An aircraft experiences several different flight phases during each flight, as shown in Figure **2**. Different flight phases can be modeled by different queuing systems separately [4, 17]. This research only focuses on modeling the en route segment. Previous research use $M/M/\infty$ queues to model the en route segment since they believe that available en route airspace is much larger than the demand under normal conditions [17]. However, this paper uses pure loss models $M(t)/M/s/s$ or $C_{m(t)}(t)/C_k/s/s$ are more accurate since the NAS has a constrained capacity, so it should be modeled similar to roadway segments [14].

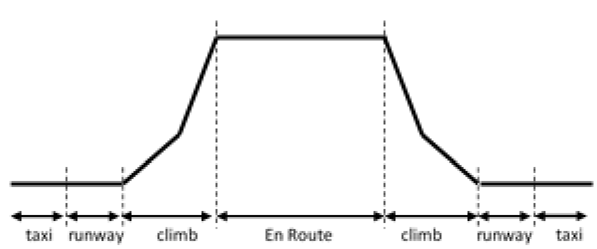


Figure 2. Flight Segments ([17],http://virtualskies.arc.nasa.gov/atm/2.html)

1.2 Overview of Time Dependent Queuing Models

For time dependent queuing models, however, solving exact numerical solutions is computationally cumbersome [18–21]. Several approximation approaches, such as Pointwise Stationary Approximation (PSA is obtained by taking the expectation of the formula for stationary performance measure with an instant arrival rate at each time point as an input parameter) [16, 22–24], the Stationary Independent Period by Period (SIPP) approach [25], and the Stationary Backlog Carryover (SBC) approach [26], Surrogate Distribution Approximation (SDA) [27, 28], Diffusion Approximation [29–32] for computing solutions of time dependent queuing models, are prevalent.

For strongly a non-stationary arrival process as in this research, segmenting the entire time period into a series of individual segment, like SIPP and SBC approaches, is a practical sensible analytical approach for the current research situation. In each segment, the arrival rate is approximated by the average arrival rate during that segment, which is used as input to a stationary queuing model. The approximation approach used in this research is also called *Piecewise Constant Coxian (Markovian) Queues*. As in the SBC approach, this also assumes that different time periods are not independent from each other. In two successive time periods, the same number of aircraft may be represented by different state space probability vectors. Consequently, a projection algorithm is developed to shift the state probability vector from one period to the next period. In addition, in contrast to the SBC and SIPP approaches, this research assumes that in each time period, steady state might not be achieved. Thus, the number of aircraft in the system (queue length), as a performance measure for both $M(t)/M/s/s$ and $C_{m(t)}(t)/C_k/s/s$ models, is calculated by averaging transient solutions over each time period. The number of aircraft in the systems as a performance measure is compared between these two types of queuing models. Both queuing models are validated by the data extracted from FACET which is driven by empirical flying data. Results show that the Markovian queue performs as well as the Coxian queue with the advantage of mathematical and computational tractability. Moreover, expected transient solutions are more accurate than steady state solutions for both types of queuing models.

1.3 Contribution

The contribution of this research is twofold. A practical approach of using piecewise constant $C_m/C_k/s/s$ to approximate time dependent queuing models $C_{m(t)}(t)/C_k/s/s$ is developed. Because the arrival process changes in each time period of the $C_{m(t)}(t)/C_k/s/s$ queue, the state spaces are different in different time periods. Consequently, this research develops a practical projection algorithm that shifts a probability state vector from a state space in one time period to that of the next period. Our second contribution includes $C_{m(t)}(t)/C_k/s/s$ and $M(t)/M/s/s$ models developed for the NAS. Our results show that $M(t)/M/s/s$ queuing models perform as well as $C_{m(t)}(t)/C_k/s/s$ models with the advantage of mathematical and computational tractability.

2 Methodology

2.1 Parameters of Pure Loss Model for NAS

As mentioned in Section 1.1, purse loss models are appropriate for modeling the NAS. A $C_{m(t)}(t)/C_k/s/s$ model and its calibration are introduced in this section.

2.1.1 Data source and assumption

FACET is a high-fidelity traffic flow simulation software package, which is driven by empirical flying data from the FAA [33]. In this research, all arrival time and service time data are extracted from FACET at 30-second intervals. The extracted data include recordings of aircraft flying information such as the location of the flight. Calibration of model parameters is based on following assumptions:

Assumption 1: It is possible to un-truncate the arrival data.

Theoretically, an arrival event can occur in 30 seconds. This research assumes arrival data is un-truncated.

Assumption 2: The set of times representing fundamental changes in the inter-arrival distribution is 24 one-hour time periods.

As mentioned in Section 1.2, the method of segmenting the entire time horizon into a series of individual segments is employed in this research. This research assumes that in every hour the arrival rate can be approximated by constant a rate, which may be different from the next hour segment.

Assumption 3: The distribution of service times does not change and remains stable for the entire time horizon.

Since the flying peed of aircraft has small variation during a day, accordingly, the time used to flying across a specific area has small variation. Thus, this research assumes service times are following time invariant distribution.

2.1.2 Capacity and Number of Service Channels

The capacity of a single airspace sector is defined by the maximum number of aircraft allowed to operate in a sector in a given time. Capacity is passed as a known parameter from upstream work, and the systematic procedure for calculating capacity is beyond the scope of this research. The maximum number of aircraft allowed in a single sector is denoted by s in the following section of this paper. The space of individual aircraft occupied in the airspace represents one queuing “server”.

2.1.3 Inter-Arrival Time and Arrival Rate

Inter-arrival time is defined as the time interval between two successive aircraft entering into an individual sector, which is recorded in FACET. If in 30 seconds there are *n (n>1)* aircraft entering into the sector, those aircraft are assumed to have arrived into the sector in equal time periods of length 30/*n*. With inter-arrival time data, we fit Exponential distributions and Coxian distributions. Since flying demand is varies during a day, a time dependent arrival rate is assumed.

Using seven days of FACET data, MATLAB code was developed to extract arrivals that include both external sources and aircraft arriving from other sectors within the network directly from FACET. Let $X_1,\ldots,X_l$ be a set of independent Coxian random variables of the inter-arrival time at different periods throughout the day. In this study, we assume l represented a one-hour time period. Then, the time dependent inter-arrival time is approximated by *a time-dependent piece-wise constant Coxian random variable* given by equation (1):

$$X(t) = \begin{cases} X_1 & t_0 < t < t_1 \\ X_2 & t_1 < t < t_2 \\ \vdots & \vdots \\ X_l & t_{l-1} < t < t_l \end{cases} \qquad (1)$$

For each time period, fitting an inter-arrival time to a Coxian distribution, the parameters of arrival rate μ_i in each phase and the continuation probabilities a_i is fitted by the Expected Maximum Likelihood Estimation (EM) algorithm [34]. The service time Coxian parameters are also found by the EM algorithm. To keep the EM algorithm computationally tractable, both inter-arrival and service time distributions are limited to three phases.

2.1.4 Service Time and Service Rate

Service time is defined as the time an aircraft takes to cross a single sector. For example, FACET records an aircraft entering into a sector at time t_1 and leaving the sector at time t_2, so service time is calculated by $t_2 - t_1$. Since the time an aircraft crosses a sector is not varying very much during a day, as a result, service times are assumed as stationary.

2.2 Background on Coxian Queues

2.2.1 Overview of Coxian Distribution

A phase-type distribution describes a random time taken for a continuous time Markov process to reach an absorbing state, where only one absorbing state exists and the stochastic process starts at a transient state. A phase-type distribution can be generalized to include many types of continuous distributions, such as Exponential distributions, Erlang distributions, Hypoexponential distributions. etc. With the advantage of approximating any non-negative continuous distribution, a phase-type distribution has the Markovian (memoryless) property. However, its generality can be problematic since it is overparameterized and parameters estimation is difficult. As a special case of a phase-type distribution, the Coxian distribution has the advantage of closely approximating any arbitrary nonnegative distribution and can overcome this problem to some extent. The Coxian distribution is a generalization of the Hypoexponential distribution in which a sequence of transient *k-1* states can enter into the absorbing state. For a *k*-phase Coxian distribution, only *2k-1* parameters need to be estimated; see Figure **3** [35, 36]. Therefore, the Coxian distribution is widely used in application, such as in the health-care industry [37–39]. The method of fitting empirical data to a phase-type distribution has also been done by researchers. Popular methods are Maximum Likelihood Estimators [40, 41], the Expectation-Maximization algorithm [34, 42], moment matching [43, 44], and other methods [45–47].

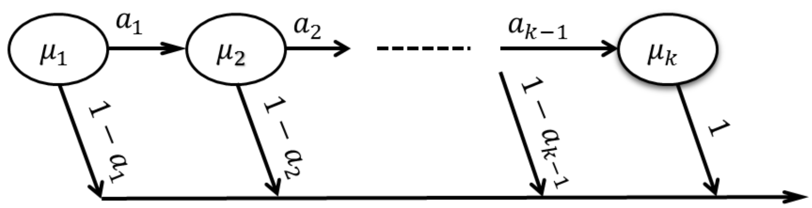


Figure 3. A *k*-phase Coxian distribution [36]

2.2.2 *Coxian Queuing Model*

As mentioned in Section 1.2, exact transient solutions of time-dependent queuing models are very cumbersome. For analyzing the transient behavior of a time dependent $C_{m(t)}(t)/C_k/s/s$ queue, the transient performance analysis of a time invariant $C_m/C_k/s/s$ queue is required. An $M(t)/M/s/s$ queue uses similar but much simpler analysis than a $C_{m(t)}(t)/C_k/s/s$ queue, so we only present an analysis for $C_{m(t)}(t)/C_k/s/s$ queues. The Champman-Kolmogorov forward equation based state enumeration is used for the solution process if the inter-arrival and service time distributions are not Exponential. The matrix notation for the Champman-Kolmogorov forward equation is as follows:

$$\dot{x}(t) = Qx(t) \tag{2}$$

where $x(t)$ is a probability vector in which the value $x_i(t)$ represents the probability of the system being in state i where $i = 1, \ldots, n$. Q is an $n \times n$ matrix called the *infinitesimal generator matrix* or the *transition rate matrix* for a Markov process. The vector $\dot{x}(t)$ represents the derivative of $x(t)$ with respect to *t*.

By integrating $\dot{x}(t) = Qx(t)$ over each time period, with the normalizing condition equation$\sum_{i=1}^{n} x(i) = 1$, the transient solution to the Markov process can be solved given an initial state probability and rate matrix *Q*. Using the matrix exponential e^{Qt}, the solution of time invariant system solution can be obtained as well by taking the limit at t approaches infinity.

The analysis of $C_m/C_k/s/s$ in each time period is nearly the same as the analysis of a queue system $C_m/C_k/s$ discussed in [48], a $C_m/C_k/s/s$ queuing system can be presented as Figure 4, where μ_{G1}, μ_{G2},..., μ_{Gm} are the arrival parameters of an *m*-phase Coxian distribution, and a_{G1}, a_{G2},...., a_{Gm} are the transition probabilities of the arrival process. We refer to this as a *generator*. Furthermore, service nodes of $C_m/C_k/s/s$ queue can be represented by a *k*-phase Coxian, with service parameters μ_{N1}, μ_{N2}, …, μ_{Nk}, and transition probabilities a_{N1}, a_{N2}, …, a_{Nk}.

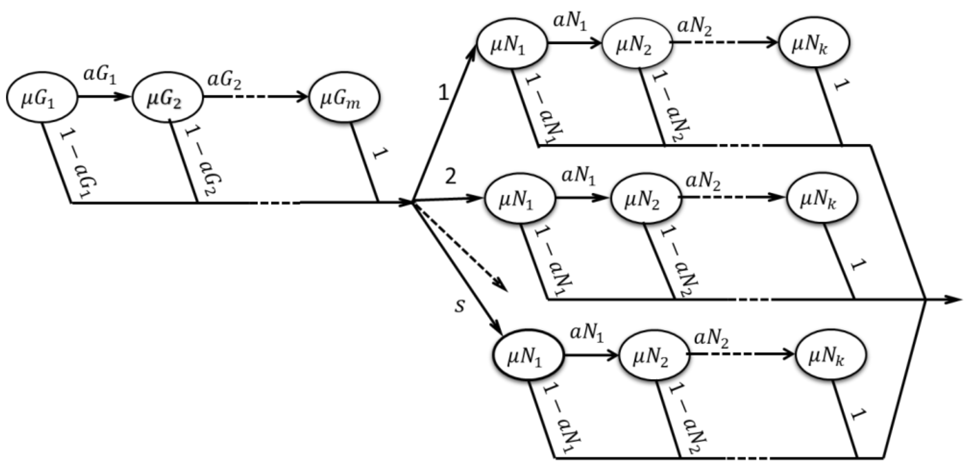


Figure 4. $C_m/C_k/s/s$ queuing system

In this study, the $C_m/C_k/s/s$ queuing system state can be defined by: (1) the phase of the arrival process or generator, (2) the number of items in service, and (3) the number of servers in each phase. Let a be the phase of the generator, let b be the number of items in service, and let c_1, c_2, …, c_k be the number of servers in phases *1, 2,…, k*, Therefore, the state of the system can be represented by the sequence $a : b(c_1, c_2, \ldots, c_k)$. The total number of servers in phases *1, 2, …, k* at a service node can be given as:

$$\sum_{i=1}^{k} c_i = b \qquad 0 \leq b \leq s, b = 0, 1, 2, \ldots \tag{3}$$

2.2.1 State Transitions

Let $N(n)$ be the total number of states for a $C_m/C_k/s/s$ queuing system in which there are at most n aircraft in the system. Then $N(n)$ is described in [48]:

$$N(n) = m\binom{n+k-1}{k} \qquad n = s \tag{4}$$

Thee dimension of transition rate matrix Q is $N(n) \times N(n)$, whose $(i,j)^{th}$ element, q_{ij}, represents the rate of transition from the j^{th} state to i^{th} state. The transitions of a $C_m/C_k/s/s$ queue described as sequence $a{:}b$ (c_1, c_2, …, c_k) above can be defined as the following events: 1. a phase change in the arrival node or generator; 2. an aircraft arrival into a service node or departure from the generator; 3. an aircraft departure from the service node; 4. a phase change in the service node. For a $C_2/C_2/3/3$ queuing system, the dimension of transition rate matrix Q is *20x20*, the rate of transition from state *1:1(1,0)* to *2:1(1,0)* is $\mu G_1 \times aG_1$; to *1:2(2,0)* is $\mu G_1 \times (1 - aG_1)$; to *1:0(0,0)* $\mu N_1 \times (1 - aN_1)$; to *1:1(0,1)* is $\mu N_1 \times aN_1$ (notations is same as in figure 2).

2.3 Time Dependent Coxian Queue

In this section, the method of using *Piece Wise Constant Coxian Queue* to approximate $C_{m(t)}(t)/C_k/s/s$ queuing is described. The transient state probability vector was determined by integrating the Champman-Kolmogorov forward equation $\dot{x}(t) = Qx(t)$ over each the time period. The approach to determine average measures of the queues is discussed. Furthermore, a projection method was developed to shift the probability vector from one period to the next period. The number of aircraft as an average measure can be obtained for each time period, which was a main metric to measure the queuing performance.

2.3.1 Probability State Vector Determination

The transition matrix $Q(t)$ can be written as given in equation (5) due to the piece-wise time dependent inter-arrival time of a $C_{m(t)}(t)/C_K/s/s$ queuing model.

$$Q(t) = \begin{cases} Q_1 & t_0 < t < t_0 \\ Q_2 & t_1 < t < t_2 \\ \vdots & \vdots \\ Q_l & t_{l-1} < t < t_l \end{cases} \tag{5}$$

This shows that the transition rate matrix varies, which can lead to the steady state probabilities not existing. However, we still would like to calculate certain average measures based upon the probabilities of each state over the time horizon using equation (6), where *M(x)* is a measure based upon the state probability vector *x*, and $\overline{M}$ is the average of the measure over time.

$$\overline{M} = \frac{\int_{t_0}^{t_1} M(x(t))dt}{t_1 - t_0} \tag{6}$$

Equation (6) can be rewritten to determine the average state probability vector $\overline{x_l}$ for each time period $t_i, \forall i = 1, \dots, l$ and an average measure $\overline{M}$ for each time period using equation (7) due to the linearity of many standard queuing model measures with respect to the state probability vector.

$$\overline{M} = \frac{\sum_{i=1}^{l}(t_i - t_{i-1}) M_i \overline{x_l}}{t_l - t_0} \tag{7}$$

How to calculate $\overline{x_l}$ for each time period $t_i, \forall i = 1, \dots, l$ will be discussed. By integrating numerically using ODE45 in MATLAB, equation (8) shows how to calculate the state probability vector over time as following:

$$x(t) = e^{Q_i(t-t_{i-1})}x(t_{i-1}), \quad \forall t \in [t_{i-1}, t_i), \forall i = 1, \dots, l \tag{8}$$

For a sufficiently small $\varepsilon > 0$, $x(t_i - \varepsilon)$ and $x(t_i)$ represent probability state vectors of two different state spaces at a given time t_i. Because this can lead to a significant calculation complication, $x(t_i), \forall i = 1, \dots, l-1$ will be redefined by two different vectors. Let $x^+(t_i)$ and $x^-(t_i)$ be the state probability vectors at time t_i in state spaces associated with periods $[t_i, t_{i+1})$ and $[t_{i-1}, t_i)$, respectively for each time $t_i, \forall i = 1, \dots, l-1$. Variables $x(t_0)$ and $x(t_l)$ are redefined as $x^+(t_0)$ and $x^-(t_l)$, as shown in equation (9), to further specify notation.

$$x^-(t_i) = e^{Q_i(t_i-t_{i-1})}x^+(t_{i-1}), \quad \forall i = 1, \dots, l \tag{9}$$

An algorithm to calculate the probability vector for each time period $\overline{x_\iota}, \forall i = 1, \dots, l$ is given by the following steps.

Step 1: Let $i = 1$ and $x^+(t_0)$ is given.

At the very beginning t_0, system is not empty. In order to give a compensation, in this research, $x^+(t_0)$ is initialized by using the transient state probability after 10 seconds from an empty system.

Step 2: Find $x^-(t_i)$ and $\overline{x_\iota}$ using integration.

Step 3: If $i < l$, then project $x^-(t_i)$ into the state space of period $[t_i, t_{i+1})$ to find $x^+(t_i)$ and go to step 2.

2.3.1.1 Projection Algorithm

The approach for projecting the probability vector $x^-(t_i)$ to vector $x^+(t_i)$ as in step 3 is shown in this section. Vectors $x^-(t_i)$ and $x^+(t_i)$ will be rewritten as $x^-$and x^- to simplify notation. The sets of the states of the inter-arrival distribution the Coxian queue associated with time periods $[t_{i-1}, t_i)$ and $[t_i, t_{i+1})$ will be defined as A^- and A^+ respectively, and the set of states of the customers in service in the Coxian queues will be defined as S. Let $x_{ij}^-(x_{ij}^+)$ be the associated component of vector $x^-(x^+)$ for each state $i \in A^-$ or $i \in A^+$ and each state $j \in S$.

The projection algorithm is described below.

Step 1: *Determine the probability state vector of the inter-arrival distribution* α^-. For each state, $i \in A^-$ set $\alpha_i^- = \sum_{j \in S} x_{ij}^-$.

Step2: *Determine the probability state vector of the inter-arrival distribution* α^+. Solve the goal programming problem in (10a-10e) in which $0 < w_1 < w_2 < \cdots$,

$$min \sum_{i=1}^{\infty}(d_i + c_i) \tag{10}$$

$$\text{s.t.}\ i!\left(\alpha^{+}(T^{+})^{-i}1\right) - i!\left(\alpha^{-}\left(T^{-}\right)^{-i}\right) + w_i d_i - w_i c_i = 0 \quad \forall i = 1,2,\ldots \tag{11}$$

$$\textstyle\sum_{i\in A^{+}} \alpha_i^{+} = 1 \tag{12}$$

$$\alpha_i^{+} \geq 0 \quad \forall i \in A^{+} \tag{13}$$

$$w_i, c_i, d_i \geq 0 \quad \forall i = 1,2,\ldots \tag{14}$$

Objective (10) minimizes the total deviation of moments between two distributions. The first constraint set -- deviation between moments of two distribution constraints (11) records the difference between moments of two distributions. Constraint (12) is the summation of initial probability state, α_i^{+} is the probability of the process starts at phase *i* and Constraints set (13) ensures it is great or equal than 0. In constraints set (14) c_i, d_i is the *i*th moments' difference, for each i, if the moment of distribution associated $[t_i, t_{i+1})$ is great than the moment of distribution associated $[t_{i-1}, t_i)$, $d_i = 0$, $c_i > 0$; otherwise $d_i \geq 0$, $c_i = 0$. w_i is the weights for *i*th moment.

Step 3: *Determine the probability state vector* x^{+}. For each state $i \in A^{+}$ and each state, $j \in S$ set $x_{ij}^{+} = \alpha_i^{+} \sum_{i\epsilon A^{-}} x_{ij}^{-}$. Once an initial probability is determined for the next period, then continue to integrate and project for its following time period.

3 Validating Time-Dependent Queuing Models with Cell-Level FACET Simulation Data

3.1 Comparison of Time Dependent Coxian and Markovian Queuing Models Results

The forms of inter-arrival times and service time distributions determine the complexity of the queuing systems. In practice, these distributions can take almost any forms in real systems. In this section, two different time dependent model results are compared. 1. The inter arrival time is a time dependent exponential distribution and the service time also an exponential distribution, which forms an *M(t)/M/s/s* queue. 2. The inter arrival time is a time dependent Coxian distribution and the service time is a Coxian distribution, which forms an $C_{m(t)}(t)/C_k/s/s$ queue. As equation (4) shows, the state space of Coxian queue increases very fast as the number of servers *s* and number of phases in the service time distribution increases. Thus, a trade off decision between accuracy and number of phases need to be made. Two and three phases Coxian distribution are widely used in previous research [39], [49]–[51]. When the Coxian queue model exceeds 3 phases, the computational time increases much longer. Thus here, both for inter-arrival and service times, the phase number are chosen between 2 to 3 phases depends on which fits data better for $C_{m(t)}(t)/C_k/s/s$ queues. In the following section, the above two types of queuing models have been validated by the cell-level spatial resolution FACET data. Results of time invariant *M/M/s/s* queues are given along with results of the aforementioned two models for comparison.

A cell is a space sector defined by specifying latitude and longitude within a 1.5-degree-by-1.5-degree square. Results for models of five cells that included the major airports ATL, DFW, JFK, LAX, and ORD are given in Figure 5 through Figure 9, which show the average number of aircraft in the system for each of the two time varying queuing models together with the time invariant queuing model. Capacity is a given constant value. The average number of aircraft in FACET simulation sample paths for June 1-7, 2007 is represented by the dotted lines, which were calculated by counting the number of aircraft at a particular cell every 30 seconds, and then averaging the total number of aircraft for every 1 hour time period. The results of the $M(t)/M/s/s$ model, and the $C_{m(t)}(t)/C_k/s/s$ model could capture the variation of demand very well in most time periods until time period 12 or 13. Investigating the data, there is a suddenly increase in the number of arriving aircraft compared to the previous period. This phenomena undermined the constant piece-wise inter arrival assumption.

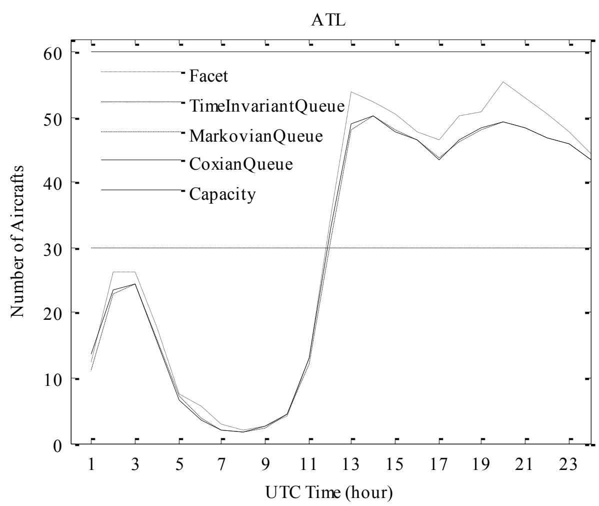


Figure 5. Number of aircraft in queuing models at ATL compared with FACET data

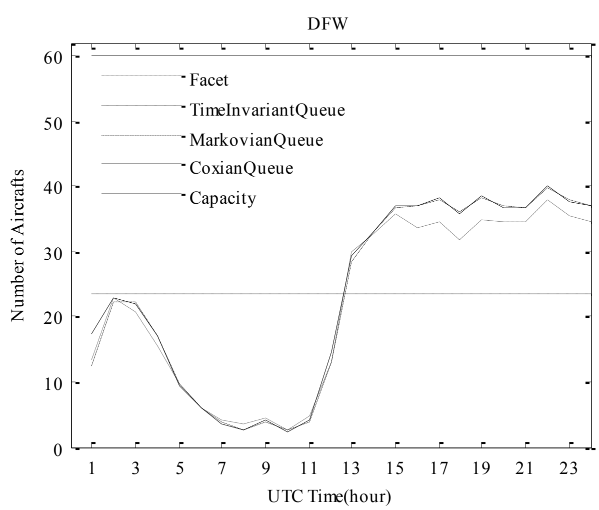


Figure 6. Number of aircraft in queuing models with different number of servers at DFW

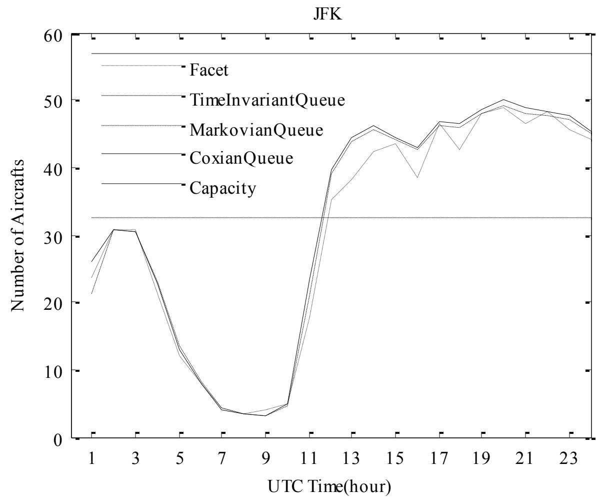


Figure 7. Number of aircraft in queuing models at JFK compared with FACET data

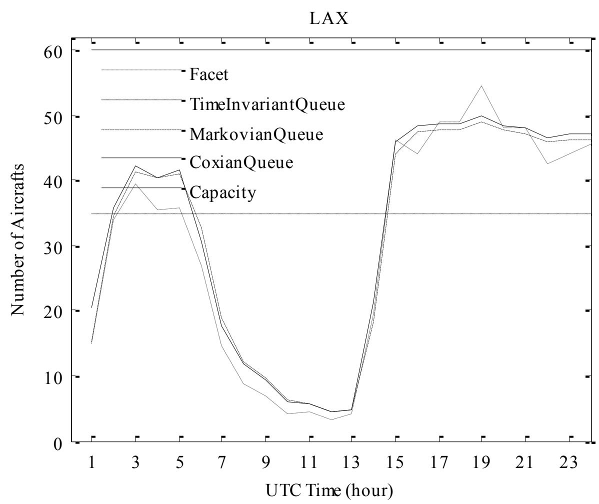


Figure 8. Number of aircraft in queuing models at LAX compared with FACET data

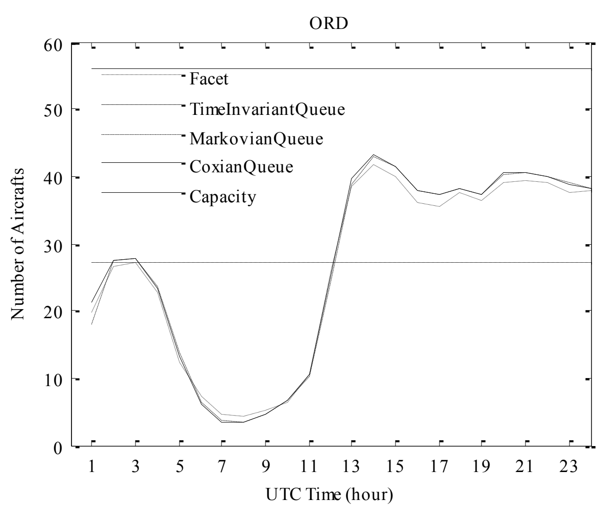


Figure 9. Number of aircraft in queuing models at ORD compared with FACET data

3.2 Comparison of Expected Transient Solution and Steady State Solution

In this section, the transient solution results are compared with steady state results of queuing models *M(t)/M/s/s* validated by Cell level data. Percentage error is the difference between model results and FACET simulation results divided by the FACET simulation results then times 100%. Average percentage error for each center is average percentage error in all 24 time period together. Comparison of average percentage error between transient solution and steady state solution in centers is shown as figure 9 and in cells is shown as figure 10. Figure 9 shows that transient solution has much more accurate approximation except in center 16, average transient solution error averages to be 8%, while that of the steady state solution is 15%. Figure 10 shows that steady state solution slightly poorer that the transient solution results. Investigating the data, service time is about 10 minutes in the cells, while it is 50 minutes in the centers. Consequently, in a one-hour time period, the cells more closely approach steady state.

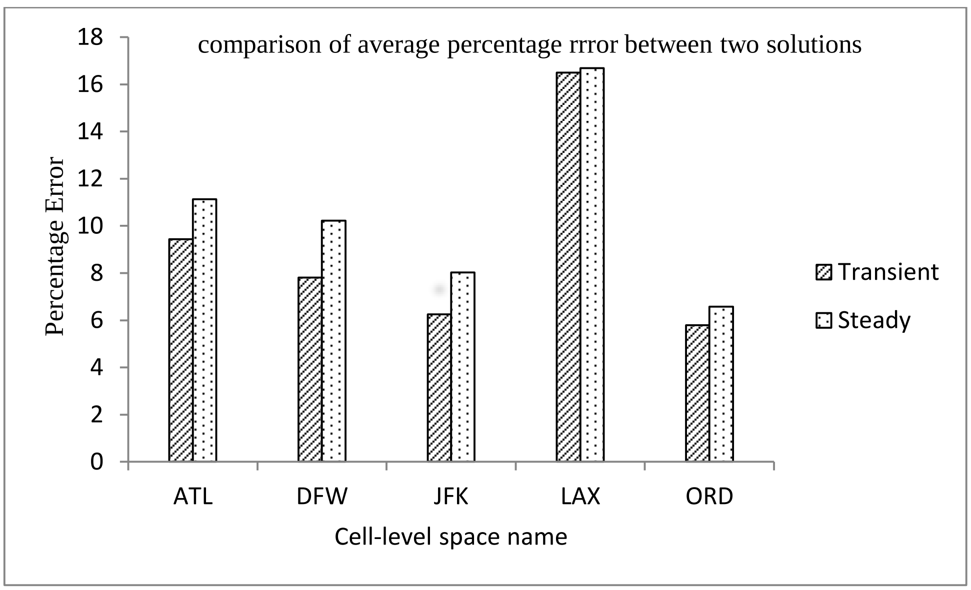


Figure 10 Average percentage error in 5 cells

4 Results and Discussion

Time dependent queue is developed in this research for solving non-stationary arrival problem. *M(t)/M/s/s* Markovian queuing model is built first, however, further statistical analysis shows inter - arrival and service times are not following exponential distribution exactly. With the advantage of closely approximating any arbitrary distribution without violating Markovian property, Coxian distribution is employed to fit both arrival time and service time distribution. Thus, $C_{m(t)}(t)/C_k/s/s$ is developed in this research as well. Solving non-stationary queuing model is time consuming. Most commonly used method is approximating non-stationary queue by stationary queue. In this research, segmenting entire period into 24 one-hour flying period. For each individual segment, stationary arrival is assumed. However, each period is not independent. Every consecutive two periods, there always exists a time epoch associated with two different probability state vectors. This research developed a projection algorithm which based on the thoughts optimization of difference between moments of two distributions. It's a natural way to shift probability state vector from one period to the next period.

Both $M(t)/M/s/s$ and $C_{m(t)}(t)/C_k/s/s$ queuing model validation results are given in Section 3. The validation results of cell-level data shows both $M(t)/M/s/s$ and $C_{m(t)}(t)/C_k/s/s$ queuing model can accurately capture variation of the demand during each day. The method of segmenting entire period into small period, and averaging over transient results of stationary queue in each time period as an estimation results successfully captured the variation of traffic demand. The validation results has two folded means: 1) Surprisingly, in each time period, $M(t)/M/s/s$ queuing model can approximate results as well as a $C_{m(t)}(t)/C_k/s/s$, however, regards to computational time $M(t)/M/s/s$ completely beats $C_{m(t)}(t)/C_k/s/s$. In another words, this research shows that, in pure loss model (waiting is not allowed) situation, Markovian model results is well as Coxian queue. 2) For each segment, the average transient results in each time period is more accurate than steady state results.

5 Acknowledgements

This research was funded by NASA Grant number NNH06ZEA001N-AS2.

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