Mixed integer linear programming approaches for land use planning that limit urban sprawl

Piyush Kumar[a], Jay Rosenberger[b], Gazi Iqbal[b,*]

[a] *TMW Systems, 899 Presidential Dr. # 117, Richardson, TX 75081*
[b] *Department of Industrial, Manufacturing, and Systems Engineering, University of Texas at Arlington, United States*

Abstract

Sprawl has a detrimental effect on quality of life and the environment. With dwindling resources and increasing populations, we must manage sprawl. Ewing et al. [1] defined factors to measure sprawl in the present urban structure. The measures are divided into four broad categories, which are density factors, mixed use factors, street factors, and center factors, and can be used in future planning of metro areas. In this research, we develop a mixed integer programming model to optimize land usage subject to sprawl constraints, which are based upon the aforementioned sprawl measures. Due to the large size of the problem, we describe a combination of heuristics and Benders' decomposition to provide an urban planner with suitable land use assignments. We show examples demonstrating how the planner can use this approach to analyze how various factors that affect land use and sprawl measures. Finally, we discuss topics of future research.

Keywords: Mixed integer linear programming, Urban planning, Sprawl, Benders' Decomposition

1. Introduction

With the industrial revolution, raw materials and finished products were needed to be delivered to the factories and to market areas. Thus, the cities needed streets, railways, shipping lanes without which the industrial revolution would have been impossible. Increased commerce and manufacturing led to congestion, new safety hazards, and air and water pollution. As the central areas became more crowded, the wealthy began moving into the suburbs. The invention of the automobile only served to hasten and promote this migration. This phenomenon was marked as an early form of urban sprawl.

1.1. Overview of Urban Planning and Methods

According to Catanese and Snyder [2], the earliest known examples of urban planning were by the Sumerians of Assyria. Their cities included fortresses and marketplaces for populations of 3000-5000 people that lived in them. The common characteristic among all of the ancient cities was that they were all built along great rivers, which afforded various advantages with regards to transportation and defense.

*Corresponding author. Tel.: +1 228 3833740
Email address: gazimddaud.iqbal@mavs.uta.edu (Gazi Iqbal)
This paper is based upon Piyush Kumar's Ph.D. dissertation.

The first example of zoning in cities was in the first century A.D. in Rome when Augustus established a 70-foot height limit. Rome struggled with the problems of overcrowding and transportation when its estimated population grew from 250,000 to 2,000,000 residents. To mitigate these problems, Romans started building roads and military cities. All of these developments in the ancient world established a pattern in which cities are now built. There are four layers in the pattern. The first one is a *physical base*, which is the visible form of the city, like the roads, buildings, and parks. This was illustrated by the rectangular pattern of the street systems. The second layer is the *political base*. For example, ancient cities were built around fortresses where the rulers of the land resided. The third layer is the *economic base*, where the planner locates various centers of commerce in the city, such as the marketplaces. The fourth layer is the *social base*, where the planner allows for open spaces or centers where the residents may assemble and socialize.

According to Catanese and Snyder [2], the major components for the urban planning process are problem diagnosis, goal articulation, prediction and projection, alternative development, feasibility analysis, evaluation, and implementation. In *problem diagnosis*, a planner must identify which problems afflict the present city, and then, define them in specific terms. However, the problem diagnosis depends on the individual planner's perspective on definitions of various norms, ideologies, and standards. Descriptive statistics is used extensively to describe a problem, such as means, medians, ranges, and ratios. An important source of information at this stage for the planner is the U.S. Bureau of Census. If the data needed by the planner are unavailable, then he/she must use survey research methods to generate specific information. After identifying the problems, specific goals must be set as to what extent the problem has to be resolved. The challenge lies in translating the verbal goals into operational objectives. The planner must determine the time span of the project. Future projections of the population growth and trade are required, since they have a direct effect on the services in the city.

After that the planner develops alternatives to the original plans. If the situation is simple, the planner has already been given a location and does not have many competing factors. But if the situation is complex and involves many different aspects, then the planner must develop multiple options. Even though the model inherently accounts for constraints, such as the size and availability of land and finance, the planner must also ask whether the alternatives are feasible on other vague constraints, such as organizational or political acceptability. As early as 1912, planners drew maps by hands of various topographical features of the land. These maps were then combined together to recommend changes in land use. This posed a problem since there was a limit to what may be feasible by hand.

One of the first places to use computers to help draw overlay maps was in Harvard in 1963. The trend continued to surge as computers became more powerful and the techniques to draw the maps became more sophisticated. The spatial data, which describe the various attributes of the land in quantifiable terms, were used as an input to optimization models. Since there are conflicting objectives when planning a city, researchers introduced decision making models where multiple criteria were evaluated. Moreno and Seigel [3] provides an application of multiple criteria evaluation via an impact analysis for the building of a highway corridor in Colorado. We examine land-use suitability analysis, which is a tool that identifies the most suitable places for locating future land uses [4].

1.2. Overview of Sprawl

As we have seen from the history of urban planning, the rise of sprawl as an issue has its roots in the Industrial Revolution. There is no consensus in the literature as to the definition of sprawl. It goes to show how difficult it is to try to measure sprawl quantitatively. There are some characteristics that are common among the many attempts to define sprawl in the literature. Those are unplanned and scattered development, low population density, high reliance on automobiles, and locations outside of the metro area. In this paper, we primarily concentrate on sprawl in the context of the United States. Delafons [5] attributes the U.S. system of urban planning to be influenced by "prairie psychology". Traditionally, development in the U.S. assumes a virtually unlimited supply of land, that land is accessible to everyone and the rights of ownership are protected by the U.S. Constitution, market driven growth is not intervened, planners do not question the need for development, and an inherent distrust towards the government and minimal public review of the policies that are already in place.

All of these social and institutional factors combined to aid urban sprawl. There are many reasons why sprawl is a cause of concern. The pace of development in the U.S. has not been proportional to the rate of population growth. For example, in the metropolitan area of Cleveland, the amount of developed area increased whereas the population decreased [6]. Loss of open space is a major contributor in prime farmland being lost to development. Low density and discontinuous development make automobile use mandatory, which results in increased usage of vehicles degrading air quality, and drivers spending on average 51 hours per year stuck in traffic [7]. Clearing land for highways, residential areas, and service areas due to sprawl lead to the destruction of green cover, which causes climate change. Sprawl leads to the destruction of the wetlands and forests, and hence, it impedes nature's ability to provide clean water.

With all of the issues surrounding sprawl, there have been past attempts to estimate the costs associated with it. One of the more significant studies done on the costs of sprawl was by Robert Burchell et al. [8, 9]. Burchell et al. [8, 9] divided the costs into five major categories: public and private capital and operating costs, transportation and travel costs, land/natural habitat preservation, quality of life, and social issues. All of the negative impacts of sprawl motivate the development of tools assisting urban planners in designing cities/downtowns that would be walk-able and transit oriented. In our research, we develop a mixed integer linear programming (MILP) model that limits the negative effects of sprawl by managing various parameters that were derived from the Transportation Research Board report by Ewing et al. [1].

The remainder of this paper is organized as follows. In Section 2, we describe related literature and the contribution of this research. Section 3 presents the MILP formulation, including a problem description, assumptions, sets, variables, and the model justification. In Section 4, we develop a Benders' decomposition algorithm to solve the MILP. In Section 5, we present an experimental set up and results. Finally, in Section 6, we discuss conclusions and future research.

2. Literature Review and Contribution

In this section, we discuss literature of land use optimization, which includes linear and integer programming techniques. Afterward, we discuss literature on measurement and optimization of sprawl, decomposition methods, and the quadratic assignment problem. Finally, we discuss the contribution of this research.

2.1. Land Use Optimization

Most literature on land use optimization models consider at least one aspect that affect sprawl. These considerations include managing peak run off, air quality, and travelling costs. The term that is frequently associated with sustainable land-use planning is smart growth. *Smart growth* refers to judicious stewardship of natural resources to prevent urban sprawl. To differentiate between the literature of simple land use allocation and sprawl, literature that explicitly mentions sprawl or sustainability as an objective are discussed in Section 2.2, while other literature on land use allocation are described in this section.

GIS-based land use suitability analysis has been used to solve an array of problems. For example, it has been used in ecological models for defining land suitability (in this case, habitat for animal and plant species [10, 11]), geological preference [12], suitability of land for agricultural use [13, 14], environmental impact evaluation [3], site selection for facilities location [15, 16], and regional planning [17]. There is also a significant part of the literature that is concerned with simultaneous optimization of land use assignment and transportation with the focus on minimizing travelling cost [18, 19, 20, 21]. Moore and Gordon [22] extend the integration of land use and transportation to include environmental applications as well. Another area of research is on optimizing the land use allocation problem with respect to economic activities [18, 23, 24, 25]. Increasing popularity of sustainability has led to research focusing on sustainable spatial optimization of land use allocation [26, 27, 28]. All of the papers cited above account for only some sprawl measures.

Most literature on land use allocation uses integer programming (IP). The decision variable of these IP models determine whether a particular activity should be allotted to a site [29]. Land use suitability analysis searches for the best site for an intended land use based on various characteristics of the land. The assumption here is that the area is subdivided into a set of basic units of observation [30]. The basic units of observation are referred to as land pieces or cells. Then, the sites are assigned a suitability factor for each category of land use, which indicates how suitable a land piece is for a particular land use.

2.1.1. Linear and Integer Programming Techniques

Implementation of linear programming (LP) models to solve land use suitability problems started with Multi-Criteria Decision Making (MCDM) techniques. MCDM involves defining a relationship between the input and output maps. The technique combines the geographical information and the planner's preferences to provide alternative decision options. After assigning weights to each objective and combining them into a single equation, the problem is solved using standard LP/IP solution approaches [31, 32]. Moore and Gordon [18] use an LP model for dividing economic activities over the planning area. They focus on how to assign the activities to a physical site in an iterative manner. Sinha [33] implements a linear optimization method and compares the resulting allocation with that of a rule based method.

2.1.2. Artificial Intelligence and Heuristic Methods

Because allocation problems are frequently huge, solution methods mostly focus on heuristic algorithms. The downside to heuristics is that they do not guarantee optimal solutions, though oftentimes, they yield near optimal solutions or sets of solutions [15]. A variety of meta-heuristic techniques, such as simulated annealing, genetic algorithms (GAs) [34], artificial neural networks [35], and cellular automata [36] are used in combination with GIS for optimization of land use allocation.

The assumptions of the input data being precise are unrealistic. With the complex factors involved in land use suitability analysis, providing accurate numerical data is challenging. Since fuzzy logic techniques have sets without clearly defined boundaries, and partial membership of elements is allowed, it works well with imprecise input data given. Wang [37] proposes a method of representing fuzzy information in GIS, which leads to the formation of a fuzzy suitability rating system. Banai et al. [38] and Jiang et al. [39] combine a fuzzy membership function with MCDM to develop GIS-based land use suitability methods. A plethora of research test the applicability of artificial neural networks for land use suitability analysis techniques [40, 41, 35]. Sui [35] uses and compares a back propagation network to measure the suitability of land pieces for development with a traditional overlay map modeling technique.

Significant papers that use evolutionary algorithms, such as GAs, to optimize multi-objective (linear or nonlinear) land use allocation problems include Brookes [42], Fotakis and Sidiropoulos [43], Holzkamper and Seppelt [44], Pereira and Duckstein [10], Matthews et al. [45], Matthews et al. [46], Los [19], Manson [47], Xiao et al. [48], Gabriel et al. [49], and Zhang and Bian [50] . Zhou and Civco [40] uses a combination of neural networks and a GA for solving a land use suitability model. Matthews et al. [45] compares GA to traditional deliberative methods. They report that the GA methods are capable of delivering a range of options, along with cost benefit analysis for each such option. Literature that explores land use optimization with simulated annealing include Bos [51] Riveira et al. [52] and Xiaoli et al. [53]. The aforementioned models strive to generate multiple solutions instead of just a single one. Hence, these models depend heavily on heuristic techniques.

2.2. Measurement and Optimization of Sprawl

In recent years, urban sprawl has been fueled by a combination of rapid economic growth and large populations in other countries. A large number of publications focus on sprawl as an issue in countries apart from the U. S. [54, 55, 56, 57, 58, 59, 60, 61]. There is a variety of research that chooses one or more aspects of sprawl to manage. Urban sprawl is minimized from the standpoint of preservation of forests and farmland [43, 44, 45, 51, 52, 53, 62]. Attempts to minimize sprawl by suggesting changes in policies at the government level have been made in the past [54, 63, 64, 65]. Gabriel et al. [49] takes a multi-objective approach to controlling sprawl in land development by considering objectives from the perspective of the government, planners, environmentalists, conservationists, and land developers. The intention of the authors is to balance the trade-off among the different objective functions. The paper employs linear and quadratic objective functions, subject to polyhedral and binary constraints, to come up with a Quadratic Mixed Integer Program (QMIP). The authors solve an example with 913 undeveloped and 4837 developed cells using XPRESS-MP solver. The measures given in Gabriel et al. [49] do not cover many measures of sprawl, such as centering factors. Stewart et al. [66] use a genetic algorithm to solve a multi-objective constrained nonlinear combinatorial programming problem. The objective functions are similar to the measures given by Ewing et al. [1], but they are generic as far as sprawl is concerned. Zielinska et al. [30, 67] develop an optimization model that minimizes perhaps the most accurate model of sprawl in the current literature. The authors suggest that having density as an objective function might result in an unsustainable solution. The paper employs a Branch-and-Bound method to solve the resulting model. They do not consider the factors that affect sprawl like mixed use development, population density, and degrees of centering. These problems involve combining the disciplines of urban planning and optimization, and it is challenging for researchers to be experts in both areas. Most attempts at optimizing land use allocation models have been made by researchers outside of the field of optimization. Zielinska et al.[26] made one of the more significant attempts at designing a sustainable land use model for urban planning, and the authors belong to the department of geography. Some of the attempts to measure sprawl quantitatively are Ewing et al. [1], Galster et al. [68], and Malpezzi [69].

We find that the current most comprehensive framework to quantify and measure sprawl is constructed by Ewing et al. [1]. Hence, we primarily focus on their measures and interpret them in a way that is suited to future land use planning. Ewing et al. [1] include 22 measures that are broadly divided into four categories, which are residential density, neighborhood mixture of homes, jobs, and services, strength of centers, such as business districts, and accessibility to the street network.

2.3. Decomposition Methods

Decomposition methods solve large-scale problems by breaking them into several smaller subproblems, along with a master problem. Dantzig-Wolfe decomposition for linear programming with angular block structure [70, 71] started the trend of decomposition of large optimization problems [72]. Some of the decomposition methods are dual methods, primal cutting plane methods, delayed column generation, and Benders' decomposition. Decomposition methods have been used in a wide variety of applications ranging from multi-commodity distribution network design [73] to locomotive and car assignment problems [74, 75, 76]. But according to the literature, decomposition methods have never been used to solve a land-use suitability problem.

2.4. Quadratic Assignment Problem

Koopmans et al. [24] introduced the concept of Quadratic Assignment Problems (QAPs) to model the problem of locating economic activities. The location of the activities depends upon the locations of other facilities in the neighborhood. Afterwards, QAPs were used to model a variety of different problems. QAPs have several formulations, such as integer linear programming (IP) formulations, mixed integer linear programming (MILP) formulations, and graph formulations [77]. As we observed, the majority of the research for QAPs tends to employ heuristic algorithms, which is a similar tendency for land use optimization. All of the quadratic formulations in land-use suitability models were solved with meta-heuristics.

2.5. Contribution

In this paper, we develop a MILP model for land use optimization. The objective of the model is to maximize suitability as well as manage sprawl. The constraints are constructed based upon the measures of sprawl given in Ewing et al. [1]. The rationale here is that various features of a metro area, such as population centers, business districts, distance to services, etc. are always present. Hence, instead of ignoring some or all of these, and maximizing suitability alone, the measures are accounted for and managed at the planning level. The contributions of this research include:

- An investigation of effects of measures on land use suitability: From the literature survey, we concluded that no other literature has attempted to study the effects of controlling bounds on various sprawl measures on the planning area. Rather, the focus has been on sustainability, which focuses on the larger context of the land use problem. We believe it over-complicates the model since destruction of farmland, pollution, and discontinuous development is a result of urban sprawl, and not the cause/characteristics. Hence, if the effects of the measures like population density are studied and understood, then that would enable the planner to make a far more educated decision with regards to future land use planning so as to minimize sprawl and still satisfy other conflicting objectives.

- Restricting the sprawl measures: Most of the research focuses on incorporating the measures in objective functions. However, as noted by Zielinska et al. [26], if population density is included as an objective function, then either maximizing or minimizing it would counter the principles of sustainable development. For example, maximizing population density would lead to overcrowding and minimizing population density would lead to sprawl. Hence, our model includes several significant measures of sprawl as constraints in the model. This allows the planner to quickly perform sensitivity analysis. It also enables the planner to generate a range of solutions based on the manipulation of the parameters.

- Use of decomposition methods: The literature is completely devoid of research that employs decomposition methods to solve large QMIPs for land use allocation, even though decomposition methods have been used extensively in other areas that involve large-scale problems. We develop a land use model with sprawl constraints and customized decomposition methods to solve it.

3. Mathematical Model

In this section, we describe measures and develop the MILP for land use.

3.1. Measures of Sprawl

Ewing et al. [1] was funded by Smart Growth America with the objective of characterizing sprawl and relating it with a wide set of outcomes. Using principal component analysis, the authors partition various sprawl factors into four categories, which are density factors, mix factors, centering factors, and street factors. *Density factors* include seven variables, four of which are measured from data by the U. S. Bureau of Census. The assumption is that census tracts that include low population density areas, such as rural tracts and deserts, are not included. These factors deal with the population density in the metro areas and their distribution. *Mix factors* are included to ensure a good mix of land uses in a compact area. Sprawl is characterized by long commuting time. For example, the principle behind measuring the percentage of residents within 1 mile of an elementary school is to minimize traveling. Hence, there should be a good mix of services for residences in an area. Metropolitan centers are considered hubs of concentrated activities that allow multi-purpose trip making, alternate modes of transport, and a sense of place in a metro area. Centers may be either residential or commercial. *Centering factors* include density gradient and coefficient of variation of population density across census tracts. Street networks in a metro area form a network, which may be dense or sparse depending on the geography and planning of the area. There is no information available regarding degree of connectedness or curvature of street networks. Hence, the authors use the information about block lengths to generate sprawl measures. *Street factors* include percentage of small blocks, average block size in square miles, and percentage of small blocks (< 0.01 square miles).

3.2. Problem Description and Assumptions

Given a set of land pieces in all or part of a metro area, the planner must assign a land use to each piece. If a land piece has a pre-existing land use, it can simply be removed from consideration or included in the model as a hard constraint. The aim of the model is to plan the area in such a way that it naturally resists sprawling in the future. To achieve this target, the planner must find a balance between population growth and services in the area. If he/she fails to do so, then the sprawl would naturally occur as we have observed from history. The planner controls bounds for the sprawl metrics given in the model. By changing these limits, the planner gets information about how the model behaves under different conditions. In some cases, the bounds also depend upon the demands of the market. In others, the bounds must be controlled to manage sprawl.

In the model, we make the following assumptions:

- There is a given finite set of land uses. For example, in this research, we consider eight different land uses, which are high industrial (HI), high commercial (HC), high industrial residential (HIR), high residential (HR), low commercial (LC), low industrial (LI), low industrial residential (LIR), and low residential (LR).

- For each land piece and each land use, the planner has already assigned a suitability value. In this research, suitability values vary from -10 to +10 depending on the fitness of the land pieces towards a land use.

- For each land piece and each land use, the planner has future population projections.
- For each pair of land pieces, the planner calculates the distances between them. In this research, we use the distances between the geographical centers of the land pieces.
- For each pair of land pieces and potential land uses, the planner calculates a measure of land mix. In this research, we assume that the measure of land mix is proportional to the sum of the suitability values and an attraction factor of the land uses, but inversely proportional to the distance between the two land pieces under consideration.
- The census tracts are known a priori and partition the set of land pieces. Census tracts are meant to be territorial units that are homogeneous with respect to factors like population characteristics, living conditions, etc. Consequently, census tracts are developed after the population has settled. However, in case of future planning, the planner may rely on clear geographical boundaries that divide the planning area into census tracts.
- The density at the center of the planning area is the density of the census tract, which includes the central coordinates of the planning area.
- For each land piece, the planner determines land pieces in a surrounding *area of influence* a priori. In this research, the area of influence includes land pieces in a 5-by-5 grid in which the center is the given land piece. This is based on the assumption that the land pieces that are outside of this area of influence have negligible effect on the mixed use factor with the given land piece.

3.3. Formulation Land Use Model

The following is a description of the sets used in the model.

- C = the set of different land uses (indexed by j).
- N = the set of land pieces in the planning area (indexed by i).
- CT = the set of census tracts in the planning area (indexed by k).
- N_k = the set of land pieces in each census tract, $k \in CT$.
- N_i = the set of land pieces within the area of influence of each land piece $i \in N$.

The parameters used in the model are as follows:

- S_{ij} = the suitability factor for each land piece $i \in N$ assigned to land use $j \in C$.

- U_j, L_j = the upper and lower bounds of land pieces that can be assigned to each land use $j \in C$.
- L_{GPD} = the lower bound on gross population density assigned to the planning area.
- U_{DG} = the upper bound on the density gradient assigned between census tracts.
- L_{Mix} = the lower bound on the land mix assigned to the planning area.
- ρ_{ij} = the estimated population for each land piece $i \in N$ assigned to land use $j \in C$.
- A_i = the area of each land piece $i \in N$.
- $AF_{j\hat{j}}$ = the attraction factor for each pair of land uses $j, \hat{j} \in C$.
- d_k = the distance between the land piece at the center of the planning area to the land piece at the center of census tract $k \in CT$.
- $d_{i\hat{i}}$ = the distance between a pair of land pieces i, $\hat{i} \in N$.
- i_0 = the land piece at the center of the planning area.
- k_0 = the census tract at the center of the planning area.
- $\omega_{ij\hat{i}\hat{j}} = \left(\frac{(S_{ij}+S_{\hat{i}\hat{j}})AF_{j\hat{j}}}{d_{i\hat{i}}}\right)$ = the land mix measure for each pair of land pieces $i, \hat{i} \in N$ assigned to land uses $j, \hat{j} \in C$, respectively.

The variables of the model are given below.

- For each land piece $i \in N$ and each land use $j \in C$, let the binary variable x_{ij} be defined such that

$$x_{ij} = \begin{cases} 1, & \text{if land piece } i \in N \text{ is assigned land use } j \in C, \\ 0, & \text{otherwise.} \end{cases}$$

- For each land piece $i \in N$, each land piece within the area of influence $\hat{i} \in N_i$, and each pair land uses $j, \hat{j} \in C$, let the binary variable $x_{ij\hat{i}\hat{j}}$ be defined such that

$$x_{ij\hat{i}\hat{j}} = \begin{cases} 1, & \text{if land pieces } i \in N \text{ and } \hat{i} \in N_i \text{ are assigned land uses } j, \hat{j} \in C, \text{ respectively,} \\ 0, & \text{otherwise.} \end{cases}$$

The mixed integer linear program (MILP) is given by the following:

$$\max \qquad z_{MILP} = \sum_{i \in N} \sum_{j \in C} S_{ij} x_{ij} \tag{1}$$

subject to:

$$\sum_{j \in C} x_{ij} = 1 \qquad \forall i \in N \tag{2}$$

$$U_j \geq \sum_{i \in N} x_{ij} \geq L_j \qquad \forall j \in C \tag{3}$$

$$\frac{\sum_{i \in N} \sum_{j \in C} \rho_{ij} x_{ij}}{\sum_{i \in N} A_i} \geq L_{GPD} \tag{4}$$

$$\sum_{\hat{i} \in N_i} \sum_{j \in C} \sum_{\hat{j} \in C} \omega_{ij\hat{i}\hat{j}} x_{ij\hat{i}\hat{j}} \geq L_{Mix} \qquad \forall i \in N \tag{5}$$

$$\frac{\sum_{i \in N_k} \sum_{j \in C} \rho_{ij} x_{ij}}{\sum_{i \in N_k} A_i} \leq \frac{\sum_{i \in N_{k_o}} \sum_{j \in C} \rho_{ij} x_{ij}}{\sum_{i \in N_{k_o}} A_i} \exp^{-d_k U_{DG}} \qquad \forall k \in CT \setminus k_o \tag{6}$$

$$x_{ij} \geq x_{ij\hat{i}\hat{j}} \qquad \forall i \in N, \hat{i} \in N_i, j, \hat{j} \in C \tag{7}$$

$$x_{\hat{i}\hat{j}} \geq x_{ij\hat{i}\hat{j}} \qquad \forall i \in N, \hat{i} \in N_i, j, \hat{j} \in C \tag{8}$$

$$x_{ij\hat{i}\hat{j}} \geq x_{ij} + x_{\hat{i}\hat{j}} - 1 \qquad \forall i \in N, \hat{i} \in N_i, j, \hat{j} \in C \tag{9}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i \in N, j, \in C \tag{10}$$

$$x_{ij\hat{i}\hat{j}} \in \{0, 1\} \qquad \forall i \in N, \hat{i} \in N_i, j, \hat{j} \in C \tag{11}$$

3.4. Model Justification

Objective (1) maximizes the overall suitability value for assigning land uses to land pieces. Constraint set (2) ensures that each land piece is assigned exactly one land use. Constraint set (3) provides the upper and lower bounds for the total number of land pieces that may have a particular land use. These equations alone represent a classical linear programming approach to optimizing a land use suitability problem [33]. Now, we add constraints to manage sprawl.

As described in Section 3.1, Ewing et al. [1] uses principal component analysis (PCA) to extract the major factors that affect sprawl and broadly classifies these factors into four major groups, which are degree of centering, density, land use mix, and street factors. Each of the variables chosen is a measure for sprawl that accounts for the greatest variation in the original dataset. The factor scores derived from the PCA are normalized to have a mean of 0 and standard variation of 1 for the sampled metropolitan areas in 2000. These values are included in Table 1 as *loading factor*. The factors with positive loading factor are those that decreases sprawl, while factors with negative loading factor increase sprawl.

Of the four major factors on sprawl in Ewing et al. [1], the MILP constrains degree of centering, density, and land use mix. Constraint set (4) allows the planner to control the gross population density of the population above a certain bound. According to Ewing et al. [1], gross population density has a loading

Table 1: Summary of measures of sprawl and corresponding loading factor

Measures of sprawl	Ewing et al. [1]	Loading Factor
Center Factors	Coefficient of variation of population density across census tracts	0.21
	Density gradient (rate of decline of density with distance from the center of the metro area)	-0.74
	Percentage of population < 3 miles from the central business district (CBD)	0.76
	Percentage of population > 10 miles from the CBD	-0.76
	Percentage of population relating to centers within the same metropolitan statistical area (MSA)	0.17
	Ratio of weighted density of population centers to highest density in the same MSA	0.48
Density Factors	Gross population density in persons per square miles (PSM)	0.89
	Percentage of population living at density < 1500 PSM	-0.69
	Percentage of population living at density > 12500 PSM	0.94
	Estimated density at the center of the metro area derived from negative exponential density function	0.90
	Gross population density of urban lands	0.94
	Weighted average lot size in square feet for single family dwellings	-0.30
	Weighted density of all population centers (local density maxima) within a metro area	0.81
Mix Factors	Percentage of residents with businesses within certain blocks of their homes	0.60
	Percentage of residents with satisfactory neighbourhood shopping within 1 mile	0.36
	Percentage of residents with schools within 1 mile	0.52
	Job-Resident balance	0.85
	Population-serving job mix	0.87
	Population serving job resident balance	0.13
Streets Factors	Approximate average block length in urbanized portion of the metro	-0.83
	Average block size in square miles (excluding blocks > 1 square mile)	-0.86
	Percentage of small blocks (< 0.01 square mile)	0.92

factor of 0.89, indicating that as it decreases, sprawl increases.

For a measure of centering, Ewing et al. [1] estimates the density at the center of the metropolitan area and the density gradient after fitting a negative exponential density function to the data points that include densities of census tracts versus the distance from the center to those census tracts. The loading factor of density gradient is -0.74, so it is limited by an upper bound using constraint set (6).

The measures for mix factor as given in Ewing et al. [1] are for metro areas that have already been developed in which schools, businesses, and shopping centers are already constructed. Because the MILP in the paper is intended for planning, we substitute these measures with another model for land mix use that is also in the literature [e.g., 67, 78, 79] . Attraction factor AF_{ij} refers to whether it is desirable to have the land uses closer together or farther apart. In constraint set (5), land mixed use factor is constrained for each land piece $i \in N$ by a lower bound using linearized quadratic variables from constraints (7)–(9).

In addition to these three major factors, Ewing et al. [1] found three measures for street factors to be significant in the PCA. However, each of them is based upon block length, which is not determined in land use planning, so street factors are not considered in the MILP.

4. Algorithm

The data-set used in the experiments was provided by the Urban Planning Department at the University of Texas at Arlington and is for the city of Leander, Texas. It has 7632 land pieces, each with a size of 150 feet by 150 feet, which are partitioned into 5 census tracts. The suitability factors for each land piece were provided for eight different categories. Considering only the assignment constraints (2), this results in a total of 8^{7632} possible land use assignments. In general, the number of variables in the MILP is $|N| \times |C| + \sum_{i \in N} |N_i| \times |C|^2$. For a 7632 land piece problem with 8 land use categories and 5 census tracts, the number of variables is roughly 4 million, and the number of constraints exceeds 12 million. Due to the large number of quadratic variable constraints, a Benders' decomposition method was chosen to solve this assignment problem.

4.1. Benders' Decomposition applied to the MILP

The MILP rapidly becomes too large for CPLEX to handle as the number of land pieces increases primarily due to the variables and constraints associated with land mixed use (5) and (7)-(9). For each neighbor of a land piece i with assigned land use j, we have a corresponding quadratic variable for each neighboring land piece $\hat{i}$ with assigned land use $\hat{j}$. Hence, for each neighboring land piece, we have 64 quadratic variables, and each quadratic variable has 3 constraints linking $x_{ij\hat{i}\hat{j}}$, x_{ij}, and $x_{\hat{i}\hat{j}}$.

Because of the large number of land mixed use variables and constraints, we revise the formulation of the MILP to penalize assignments that violate the land mixed use constraints (5), instead of maintaining them as hard constraints as in the MILP. Let λ be a positive constant penalty, and for each $i \in N$, let s_i be the violation of the lower bound L_{Mix} for the associate constraint in set (5). This penalized reformulation (MILP-p) is as follows:

$$\max \quad z_{MILP-p} = \sum_{i \in N} \sum_{j \in C} S_{ij} x_{ij} - \lambda \sum_{i \in N} s_i \tag{12}$$

$$\text{subject to:} \quad (2)-(4), (6)-(11)$$

$$-s_i - \sum_{j \in C} \sum_{\hat{i} \in N_i} \sum_{\hat{j} \in C} \omega_{ij\hat{i}\hat{j}} x_{ij\hat{i}\hat{j}} \leq -L_{Mix} \qquad \forall i \in N \tag{13}$$

$$s_i \geq 0 \qquad \forall i \in N \tag{14}$$

MILP-p can also be decomposed using Benders reformulation in which the master problem finds assignments, and the subproblem determines the land mixed use penalty. Specifically, let $\bar{x}$ be an assignment vector from the master problem. The formulation of the primal subproblem (PS) is given below:

$$\max \qquad z_{PS} = -\sum_{i \in N} s_i \tag{15}$$

subject to: (14)

$$-s_i - \sum_{j \in C} \sum_{\hat{j} \in C} \left(\sum_{\hat{i} \in N_i : \hat{i} > i} \omega_{ij\hat{i}\hat{j}} x_{ij\hat{i}\hat{j}} + \sum_{\hat{i} \in N_i : \hat{i} < i} \omega_{\hat{i}\hat{j}ij} x_{\hat{i}\hat{j}ij} \right) \leq -L_{Mix} \qquad \forall i \in N \tag{16}$$

$$x_{ij\hat{i}\hat{j}} \leq \bar{x}_{ij} \qquad \forall i \in N, \hat{i} \in N_i, i < \hat{i}, j, \hat{j} \in C \tag{17}$$

$$x_{ij\hat{i}\hat{j}} \leq \bar{x}_{\hat{i}\hat{j}} \qquad \forall i \in N, \hat{i} \in N_i, i < \hat{i}, j, \hat{j} \in C \tag{18}$$

$$-x_{ij\hat{i}\hat{j}} \leq 1 - \bar{x}_{ij} - \bar{x}_{\hat{i}\hat{j}} \qquad \forall i \in N, \hat{i} \in N_i, i < \hat{i}, j, \hat{j} \in C \tag{19}$$

$$x_{ij\hat{i}\hat{j}} \geq 0 \qquad \forall i \in N, \hat{i} \in N_i, i < \hat{i}, j, \hat{j} \in C \tag{20}$$

Observe that with a solution from the master problem $\bar{x}$, the primal subproblem yields solutions with integer values for x, so constraints in set (11) can be relaxed as in (20). In addition, this formulation of PS takes advantage of the fact that only a single quadratic variable is needed for each variable assignment pair, instead of two as suggested in MILP and MILP-p.

To formulate the dual subproblem, let π, μ^{I}, μ^{II}, and μ^{III} be dual variable vectors corresponding to constraints (16), (17), (18), and (19), respectively. The formulation of dual subproblem is given below:

$$\min z_{DS} = -L_{Mix} \sum_{i \in N} \pi_i + \sum_{i \in N} \sum_{\hat{i} \in N_i : \hat{i} > i} \sum_{j \in C} \sum_{\hat{j} \in C} \bar{x}_{ij} \mu^{I}_{ij\hat{i}\hat{j}} + \bar{x}_{\hat{i}\hat{j}} \mu^{II}_{ij\hat{i}\hat{j}} + (1 - \bar{x}_{ij} - \bar{x}_{\hat{i}\hat{j}}) \cdot \mu^{III}_{ij\hat{i}\hat{j}} \tag{21}$$

subject to:

$$-\omega_{ij\hat{i}\hat{j}}(\pi_i + \pi_{\hat{i}}) + \mu^{I}_{ij\hat{i}\hat{j}} + \mu^{II}_{ij\hat{i}\hat{j}} - \mu^{III}_{ij\hat{i}\hat{j}} \geq 0 \qquad \forall i \in N, \hat{i} \in N_i, i < \hat{i}, j, \hat{j} \in C \tag{22}$$

$$1 \geq \pi_i \geq 0 \qquad \forall i \in N \tag{23}$$

$$\mu^{I}_{ij\hat{i}\hat{j}}, \mu^{II}_{ij\hat{i}\hat{j}}, \mu^{III}_{ij\hat{i}\hat{j}} \geq 0 \qquad \forall i \in N, \hat{i} \in N_i, i < \hat{i}, j, \hat{j} \in C \tag{24}$$

Let Π be the extreme points of the polyhedron represented by constraints (22)–(24), and for each $(\bar{\pi}, \bar{\mu}) \in \Pi$, let $z(\bar{\pi}, \bar{\mu})$ be the objective value of the primal and dual subproblems. Let θ be an upper bound on the subproblem objective function. The master problem is the following:

$$\max \quad z_{RMP} = \sum_{i \in N} \sum_{j \in C} S_{ij} x_{ij} + \lambda \theta \tag{25}$$

subject to: $(2) - (4), (5), (6), (10)$

$$\theta \leq z(\bar{\pi}, \bar{\mu}) + \sum_{i \in N} \sum_{j \in C} \left[\sum_{\hat{j} \in C} \left(\sum_{\hat{i} \in N_i : \hat{i} > i} (\bar{\mu}^{I}_{ij\hat{i}\hat{j}} - \bar{\mu}^{III}_{ij\hat{i}\hat{j}}) + \sum_{\hat{i} \in N_i : \hat{i} < i} (\bar{\mu}^{II}_{\hat{i}\hat{j}ij} - \bar{\mu}^{III}_{\hat{i}\hat{j}ij}) \right) x_{ij} \right] \qquad \forall (\bar{\pi}, \bar{\mu}) \in \Pi \tag{26}$$

$$\theta \text{ is free} \tag{27}$$

In the Benders decomposition algorithm, given by Algorithm 1, a restricted master problem (RMP) is iteratively solved over a subset of the dual extreme points $\bar{\Pi} \subset \Pi$.

Set STOP = FALSE, $\theta = -\infty$, $\bar{\Pi} = \emptyset$;

Solve a relaxed problem (1)–(3), (10) to get land use assignment $\bar{x}$.;

while *STOP = FALSE* **do**

  Solve the dual subproblem (21)–(24) to get dual extreme point $(\bar{\pi}, \bar{\mu})$ and land mix violation $z(\bar{\pi}, \bar{\mu})$;

  if $\theta = z(\bar{\pi}, \bar{\mu})$ **then**

    STOP = TRUE;

  else

    $\bar{\Pi} = \bar{\Pi} \cup (\bar{\pi}, \bar{\mu})$;

  end

  Solve the RMP (2) – (4), (5), (6), (10), (25), (27), over the subset of dual extreme points $\bar{\Pi}$ in constraint (26);

end

Algorithm 1: Benders' Decomposition Algorithm

4.2. Solving the Subproblem

The quadratic assignment variable $x_{ij\hat{i}\hat{j}}$ is dependent on two binary assignment variables, $\bar{x}_{ij}$ and $\bar{x}_{\hat{i}\hat{j}}$, from the RMP. There are 4 possible combinations for the two binary variables. Consider the following primal-dual solution. For each $i \in N$, let dual variable $\bar{\pi}_i$ be such that

$$\bar{\pi}_i = \begin{cases} 1, & \text{if } L_{Mix} > \sum_{j \in C} \sum_{\hat{j} \in C} \left(\sum_{\hat{i} \in N_i : \hat{i} > i} \omega_{ij\hat{i}\hat{j}} \bar{x}_{ij} \bar{x}_{\hat{i}\hat{j}} + \sum_{\hat{i} \in N_i : \hat{i} < i} \omega_{\hat{i}\hat{j}ij} \bar{x}_{ij} \bar{x}_{\hat{i}\hat{j}} \right), \\ 0, & \text{otherwise.} \end{cases} \tag{28}$$

The dual variables $\bar{\mu}^{I}$, $\bar{\mu}^{II}$, and $\bar{\mu}^{III}$ can be constructed by the following 4 cases:

Case I: If $\bar{x}_{ij} = 0$ and $\bar{x}_{\hat{i}\hat{j}} = 0$, which implies $x_{ij\hat{i}\hat{j}} = 0$, then

$$\bar{\mu}^{I}_{ij\hat{i}\hat{j}} = \max(\omega_{ij\hat{i}\hat{j}}(\bar{\pi}_i + \bar{\pi}_{\hat{i}}), 0), \quad \bar{\mu}^{II}_{ij\hat{i}\hat{j}} = 0, \quad \bar{\mu}^{III}_{ij\hat{i}\hat{j}} = 0.$$

Case II: If $\bar{x}_{ij} = 0$ and $\bar{x}_{\hat{i}\hat{j}} = 1$, which implies $x_{ij\hat{i}\hat{j}} = 0$, then

$$\bar{\mu}^{I}_{ij\hat{i}\hat{j}} = \max(\omega_{ij\hat{i}\hat{j}}(\bar{\pi}_i + \bar{\pi}_{\hat{i}}), 0), \quad \bar{\mu}^{II}_{ij\hat{i}\hat{j}} = 0, \quad \bar{\mu}^{III}_{ij\hat{i}\hat{j}} = 0.$$

Case III: If $\bar{x}_{ij} = 1$ and $\bar{x}_{\hat{i}\hat{j}} = 0$, which implies $x_{ij\hat{i}\hat{j}} = 0$, then

$$\bar{\mu}^{I}_{ij\hat{i}\hat{j}} = 0, \quad \bar{\mu}^{II}_{ij\hat{i}\hat{j}} = \max(\omega_{ij\hat{i}\hat{j}}(\bar{\pi}_i + \bar{\pi}_{\hat{i}}), 0), \quad \bar{\mu}^{III}_{ij\hat{i}\hat{j}} = 0.$$

Case IV: If $\bar{x}_{ij} = 1$ and $\bar{x}_{\hat{i}\hat{j}} = 1$, which implies $x_{ij\hat{i}\hat{j}} = 1$, then

$$\bar{\mu}^{I}_{ij\hat{i}\hat{j}} = \max(\omega_{ij\hat{i}\hat{j}}(\bar{\pi}_i + \bar{\pi}_{\hat{i}}), 0), \quad \bar{\mu}^{II}_{ij\hat{i}\hat{j}} = 0, \quad \bar{\mu}^{III}_{ij\hat{i}\hat{j}} = \max(-\omega_{ij\hat{i}\hat{j}}(\bar{\pi}_i + \bar{\pi}_{\hat{i}}), 0)$$

Moreover, Proposition 1 shows that the dual solution constructed by (28) and cases I-IV is optimal.

Proposition 1. *A dual solution $(\bar{\pi}, \bar{\mu})$ as constructed by (28) and cases I-IV is optimal for the dual sub-problem.*

Proof. $(\bar{\pi}, \bar{\mu})$ satisfy by the dual constraints (22) - (24) by construction. Consider a primal solution $(\tilde{x}, \tilde{s})$ constructed as follows $\forall i \in N, \hat{i} \in N_i, i < \hat{i}, j, \hat{j} \in C$, $\tilde{x}_{ij\hat{i}\hat{j}} = \bar{x}_{ij}\ \bar{x}_{\hat{i}\hat{j}}$, and $\forall i \in N$.

$$\tilde{s}_i = \max\left[L_{Mix} - \sum_{j \in C}\sum_{\hat{j} \in C}\left(\sum_{\hat{i} \in N_i : \hat{i} > i} \omega_{ij\hat{i}\hat{j}}\bar{x}_{ij}\bar{x}_{\hat{i}\hat{j}} + \sum_{\hat{i} \in N_i : \hat{i} < i} \omega_{\hat{i}\hat{j}ij}\bar{x}_{ij}\bar{x}_{\hat{i}\hat{j}}\right), 0\right] \tag{29}$$

$(\tilde{x}, \tilde{s})$ satisfies the primal constraints (14), (16) - (20) by construction. What remains to be shown is that $(\bar{\pi}, \bar{\mu})$ and $(\tilde{x}, \tilde{s})$ are complementary optimal solutions. Consider the following complementary slackness conditions,

$$\bar{\pi}_i\left(s_i + \sum_{j \in C}\sum_{\hat{j} \in C}\left(\sum_{\hat{i} \in N_i : \hat{i} > i} \omega_{ij\hat{i}\hat{j}}\bar{x}_{ij}\bar{x}_{\hat{i}\hat{j}} + \sum_{\hat{i} \in N_i : \hat{i} < i} \omega_{\hat{i}\hat{j}ij}\bar{x}_{ij}\bar{x}_{\hat{i}\hat{j}}\right) - L_{Mix}\right) = 0 \qquad \forall i \in N \tag{30}$$

$$\bar{\mu}^{I}_{ij\hat{i}\hat{j}}(\bar{x}_{ij} - \tilde{x}_{ij\hat{i}\hat{j}}) = 0 \qquad \forall i \in N, \hat{i} \in N_i, i < \hat{i}, j, \hat{j} \in C \tag{31}$$

$$\bar{\mu}^{II}_{ij\hat{i}\hat{j}}(\bar{x}_{\hat{i}\hat{j}} - \tilde{x}_{ij\hat{i}\hat{j}}) = 0 \qquad \forall i \in N, \hat{i} \in N_i, i < \hat{i}, j, \hat{j} \in C \tag{32}$$

$$\bar{\mu}^{III}_{ij\hat{i}\hat{j}}(\tilde{x}_{ij\hat{i}\hat{j}} + 1 - \bar{x}_{ij} - \bar{x}_{\hat{i}\hat{j}}) = 0 \qquad \forall i \in N, \hat{i} \in N_i, i < \hat{i}, j, \hat{j} \in C \tag{33}$$

$$\tilde{x}_{ij\hat{i}\hat{j}}(-\omega_{ij\hat{i}\hat{j}}(\bar{\pi}_i + \bar{\pi}_{\hat{i}}) + \bar{\mu}^{I}_{ij\hat{i}\hat{j}} + \bar{\mu}^{II}_{ij\hat{i}\hat{j}} - \bar{\mu}^{III}_{ij\hat{i}\hat{j}}) = 0 \qquad \forall i \in N, \hat{i} \in N_i, i < \hat{i}, j, \hat{j} \in C \tag{34}$$

$$\tilde{s}_i(1 - \bar{\pi}_i) = 0 \qquad \forall i \in N \tag{35}$$

Conditions (30) and (35) follow from (28) and (29). In cases I, II, and IV, $\bar{x}_{ij} = \tilde{x}_{ij\hat{i}\hat{j}}$, and $\bar{\mu}^{I}_{ij\hat{i}\hat{j}} = 0$ in case III, which implies (31). Similarly, $\bar{x}_{ij} = \tilde{x}_{ij\hat{i}\hat{j}}$ in cases I, III, and IV, and $\bar{\mu}^{II}_{ij\hat{i}\hat{j}} = 0$ for case II, so conditions (32) hold. (33) follows from the fact that $\tilde{x}_{ij\hat{i}\hat{j}} = \bar{x}_{ij} + \bar{x}_{\hat{i}\hat{j}} - 1$ in case IV, and $\bar{\mu}^{III}_{ij\hat{i}\hat{j}} = 0$ in cases I-III. For (34), $\tilde{x}_{ij\hat{i}\hat{j}} = 0$ in case I-III, and we have two subcases for case IV.

Sub case a: For $\omega_{ij\hat{i}\hat{j}} \geq 0$, $\bar{\mu}^{I}_{ij\hat{i}\hat{j}} = \omega_{ij\hat{i}\hat{j}}(\bar{\pi}_i + \bar{\pi}_{\hat{i}})$, $\bar{\mu}^{II}_{ij\hat{i}\hat{j}} = \bar{\mu}^{III}_{ij\hat{i}\hat{j}} = 0$.

Sub case b: For $\omega_{ij\hat{i}\hat{j}} < 0$, $\bar{\mu}^{III}_{ij\hat{i}\hat{j}} = -\omega_{ij\hat{i}\hat{j}}(\bar{\pi}_i + \bar{\pi}_{\hat{i}})$, $\bar{\mu}^{I}_{ij\hat{i}\hat{j}} = \bar{\mu}^{II}_{ij\hat{i}\hat{j}} = 0$.

Hence, conditions (30)–(35) are satisfied, so the proof is complete. □

The benefit of this primal-dual solution method is that the coefficients of the Benders cut (26) can be calculated without formulating the dual subproblem or even storing the values of $\bar{\mu}$ in memory.

5. Experimental Setup and Results

5.1. Experimental Setup

The goals of the experiment are to create an efficient frontier between land suitability and land mixed use violation and to obtain the best possible solution in a limited time. Initially, a relaxed problem, (1)–(2), (10), which only maximizes land use suitability while ignoring the explicit sprawl constraints, is solved. Once we obtain an optimal solution to the relaxed problem, the gross population density and average density gradient are calculated. Based on these values, a central composite design was used to design the experiment to show how a planner could decide what the bounds on various constraints should be. Characteristics of the experimental setup are discussed below.

The time limit on the CPLEX optimizer for the master problem is 30 minutes, and the time limit on Benders' algorithm overall is 5 hours. The parameter in CPLEX for MIP emphasis was set to feasibility instead of optimality for the master problem. If the gap between subproblem objective value and master problem parameter is less than 0.1, the Benders' decomposition algorithm terminates.

With a sufficiently large penalty value λ, MILP-p and MILP are equivalent problems. However, from a practical urban planning perspective land use and land mix are both objectives that a planner would like to consider. Consequently, a planner would likely specify values for λ and L_{Mix} based upon his/her preferences in practice.

A 3 – factorial design was used to collect observations. The constraints for land use categories in set (3) were relaxed, and the penalty value λ was set to 1. Given the value of gross population density for the planning area from solving the relaxed model, the lower bound on the gross population density, L_{GPD}, was increased by 20 people per square mile. The increase in lower bound by 20 people per square mile was decided upon by trial and error. If the lower bound on the gross population density is increased by a smaller amount, it does not have a significant effect on the solution. If the lower bound is varied by a larger amount, it led to infeasibility. Given the average value of the density gradient from solving the relaxed model, the upper bounds, U_{DG}, in the experiments were obtained by reducing the relaxed value by 3.5 units successively. The decrease in upper bound for density gradient was determined at the same time when searching for the variation on lower bound for gross population density.

5.2. Results

Table 2 shows the results for the aforementioned problem for Leander, Texas, with 7632 land pieces. Table 2 contains $3^3 + 1 = 28$ data points, which is as a result of all possible unique combinations of three

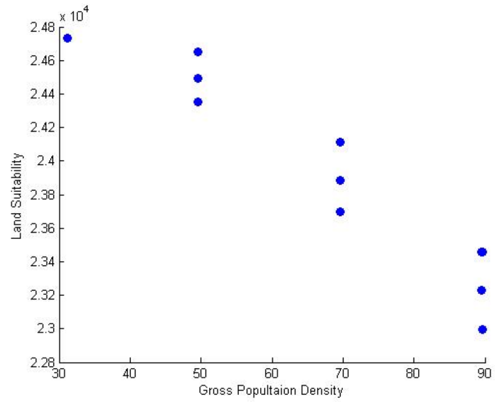
Table 2: Results from 7632 land pieces

Sprawl Bounds			Solution				CPU Time (seconds)	No. of Bender's Cuts
L_{Mix}	L_{GPD}	U_{DG}	Land Suitability	Land Mixed Use Penalty	GPD (Persons / sq. mile)	Avg. Density Gradient		
N/A	N/A	N/A	24 733	0	31.125	1.994	0	0
0.005	49.57	−0.37	24 652	−0.041	49.621	−4.91	0	0
0.01	49.57	−0.37	24 652	−0.445	49.621	−4.91	0	1
0.015	49.57	−0.37	24 652	−1.713	49.612	−4.698	420	70
0.005	49.57	−3.87	24 495	−0.046	49.585	−8.342	0	0
0.01	49.57	−3.87	24 495	−0.402	49.585	−8.342	0	2
0.015	49.57	−3.87	24 495	−1.857	49.585	−8.342	>18 000	406
0.005	49.57	−7.37	24 354	−0.059	49.596	−13.183	0	0
0.01	49.57	−7.37	24 354	−0.521	49.596	−13.183	0	5
0.015	49.57	−7.37	24 354	−1.964	49.596	−13.183	>18 000	399
0.005	69.57	−0.37	24 113	−0.081	69.596	−0.865	0	0
0.01	69.57	−0.37	24 114	−0.425	69.57	−0.929	60	13
0.015	69.57	−0.37	24 114	−1.899	69.578	−0.862	13 260	442
0.005	69.57	−3.87	23 885	−0.096	69.587	−4.951	0	0
0.01	69.57	−3.87	23 886	−0.508	69.569	−4.96	60	11
0.015	69.57	−3.87	23 885	−1.831	69.587	−5.001	>18 000	417
0.005	69.57	−7.37	23 699	−0.037	69.593	−15.225	0	2
0.01	69.57	−7.37	23 699	−0.488	69.593	−15.224	60	14
0.015	69.57	−7.37	23 699	−1.984	69.593	−15.205	>18 000	376
0.005	89.57	−0.37	23 458	−0.038	89.593	−2.48	0	2
0.01	89.57	−0.37	23 459	−0.593	89.575	−2.496	180	38
0.015	89.57	−0.37	23 458	−2.351	89.593	−2.502	>18 000	322
0.005	89.57	−3.87	23 230	−0.038	89.583	−6.784	0	2
0.01	89.57	−3.87	23 230	−0.58	89.583	−6.773	>18 000	486
0.015	89.57	−3.87	23 230	−2.793	89.583	−6.76	>18 000	310
0.005	89.57	−7.37	22 997	−0.219	89.587	−13.021	0	2
0.01	89.57	−7.37	22 997	−1.091	89.587	−13.021	360	66
0.015	89.57	−7.37	22 997	−3.681	89.587	−13.021	>18 000	486

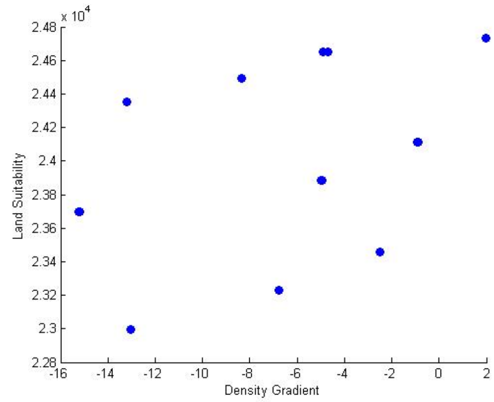
factors at three different levels, and the first experiment is the results from solving the relaxed problem used to find a maximum land use suitability value. The table shows how values of the factors from their respective default values change. In addition to the results displayed in Table 2, we conducted experiments using a solution pool when solving the RMP, generating multiple Benders' cuts per iteration. However, the solutions from using multiple Benders' cuts did not improve the solutions, and CPU times were typically larger than those presented here.

From Table 2, we can see, the CPU time is highly dependent upon the lower bound of the land mixed use. In most cases in which the lower bound is tightened to 0.015, the 5-hour time limit elapses prior to finding a provably optimal land use assignment. This is primarily because the bound on the land mixed use constraints increases the number of cuts generated from the subproblem. The average density gradients are rarely at their upper bounds. This is primarily due to the differences of the density gradients across the census tracts. Tightening the bounds on gross population density and density gradient increases the urban areas and population within them, which decreases sprawl but also reduces land suitability. However, tightening bounds on land mix use has only small changes in planning solutions.

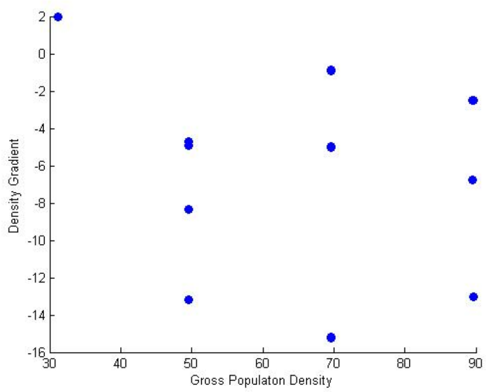
Figure 1 shows scatter plots for land suitability versus gross population density, land suitability versus average density gradient, and average density gradient versus gross population density. From figure 1a, we can see that a decrease in gross population density results in an increase in land suitability. However, Table 2 shows that an increase in gross population density slightly worsens land mixed use. This behavior leads us to conclude that gross population density has a very clear linear inverse relationship with land suitability,



(a) Land Suitability vs. Gross Population Density



(b) Land Suitability vs. Density Gradient



(c) Density Gradient vs. Gross Population Density

Figure 1: Scatter Plot

while having a minimal effect on land mixed use. In figure 1b, we observe that decreases in average density gradient result in decreases in the land suitability. In addition, Table 2 shows that, unlike gross population density, decreases in density gradient increase land mixed use violations. Finally, figure 1c shows that gross population density and density gradient are only slightly positively correlated.

6. Conclusions and Future Research

Urban sprawl is a genuine problem in all the major cities of the world. Controlling urban sprawl would make the cities sustainable and pleasant places to live. Given the various sprawl factors defined by Ewing et al. [1], we formulated a mixed integer linear programming (MILP) model for urban land use assignment with the focus on controlling urban sprawl. The MILP model was then solved using Benders' decomposition. The subproblem was solved using a deterministic method that employed properties from duality theory instead of solving it using a commercial solver. Since the problem has a number of factors affecting urban sprawl, the sprawl constraints were introduced as bounds instead of putting them in the objective function. We then created scatter plots comparing these factors and land suitability. Such a scatter plot allows a planner to analyze the effects of various factors on land suitability. This would assist the planner in determining the best land use assignment for a given area.

There are a number of factors that affect sprawl. Even using 3 factors over 3 different levels yields 27 different planning problems. Hence, as the number of factors increases, the number of planning problems increases exponentially. Thus, given the amount of time it takes to solve the MILP, there is a limit on the number of factors that can be incorporated into an experiment.

In our experiments, the time taken to solve the master problem was negligible, whereas the time to generate Benders' cuts was very large, consuming most of the CPU time. The reason for the large time consumption is that while solving the subproblem, we generate the violations in land mixed use, create the dual subproblem objective coefficients, and then recombine them to form a Benders' cut. In all of these steps, the index for the variables depend on four dimensions, which are land piece i, land piece $\hat{i}$, land use category j and land use category $\hat{j}$. The time taken to search over these four dimensions is very long. One solution is to form a sparse four- dimensional matrix but that would be very expensive memory-wise. Hence, the goal is to find a way to calculate the values which is efficient with respect to both computations and memory. It would reduce the time to generate the Benders' cuts. This would also enable the inclusion of more quadratic variables. Hence, in future research, various constraints are being studied to isolate quasi-independent factors that can then be used in the orthogonal design.

Acknowledgements

We would like to thank Dr. Ardeshir Anjomani at the University of Texas at Arlington and Dr. Parmanand Sinha at the University of Tennessee for providing data and additional domain expertise in urban planning with this research.

References

[1] R. Ewing, R. Pendal, D. Chen, Measuring sprawl and its impact, Tech. rep., Smart Growth America, Washington, D.C (2002).

[2] A. J. Catanese, J. C. Snyder (Eds.), Introduction to Urban Planning, McGraw-Hill, New York, 1979.

[3] D. Moreno, M. Seigel, A GIS approach for corridor siting and environmental impact analysis, in: GIS/LIS'88 Proceedings from the Third Annual International Conference, San Antonio, Texas, 1988, pp. 507–514.

[4] M. G. Collins, F. R. Steiner, M. J. Rushman, Land-use suitability analysis in the United States: historical development and promising technological achievements, Environmental Management 28 (5) (2001) 611–21.

[5] J. Delafons (Ed.), Land-use Controls in the United States, MIT Press, Cambridge, MA, 1962.

[6] F. K. Benfield, D. R. Matthew, D. D. T. Chen, Once there were greenfields: How urban sprawl is undermining america's environment, economy, and social fabric, Tech. rep., Natural Resources Defense Council with Surface Transportation Policy Project,New York (1999).

[7] D. Howlett, Study finds traffic congestion bad and getting worse, USA Today (October 1 2003).

[8] R. Burchell, The costs of sprawl-revisited, Tech. rep., National Academy Press, Washington DC (1998).

[9] R. Burchell, The costs of sprawl-2000, Tech. rep., National Academy Press, Washington DC (2002).

[10] J. M. C. Pereira, L. Duckstein, A multiple criteria decision making approach to GIS-based land suitability evaluation, International Journal of Geographical Information Systems 7 (5) (1993) 407–424.

[11] R. Store, J. Kangas, Integrating spatial multi-criteria evaluation and expert knowledge for GIS-based habitat suitability modelling, Landscape and Urban Planning 55 (2) (2001) 79–93.

[12] G. Bonham-Carter (Ed.), Geographic Information Systems for Geoscientists: Modeling with GIS, Pergamon Press, Oxford, UK, 1994.

[13] J. C. Cambell, J. Radke, J. T. Gless, R. M. Whirtshafter, An application of linear programming and geographic information systems: Cropland allocation in antigue, Environment and Planning A 24 (1992) 535–549.

[14] S. Kalogirou, Expert systems and GIS: an application of land suitability evaluation, Computers, Environment and Urban Systems 26 (2–3) (2002) 89–112.

[15] J. R. Eastman (Ed.), GIS and Decision Making, UNITAR, Geneva, 1993.

[16] R. L. Church, Geographical information systems and location science, Computers and Operations Research 29 (6) (2002) 541–562.

[17] Multicriteria Analysis and Geographical Information Systems: An Application to Agricultural Land Use in Netherlands, Kluwer Academic Publishers, 1990.

[18] J. E. Moore II, P. Gordon, A sequential programming model of urban land development, Socio-economic Planning Sciences 24 (3) (1990) 199–216.

[19] M. Los, Simultaneous optimization of land use and transportation, Regional Sciences and Urban Economics 8 (1978) 21–42.

[20] L. Q. Ouyang, W. H. K. Lam, An activity based land use and transportation optimization model, Journal of the Eastern Asia Society for Transportation Studies 8.

[21] A. Vold, Optimal land use and transport planning for the greater Oslo area, Transportation Research Part A 39 (2005) 548–565.

[22] J. E. Moore II, T. J. Kim, Mill's urban system models: Perspective and template for lute (land use/transport/environment) applications, in: 7th World Transportation Conference SIG-1, Sydney, NSW, Australia, 1995.

[23] M. Beckmann (Ed.), Studies in the Economics of Transportation, Yale University Press, New Haven, CT, 1956.

[24] T. C. Koopmans, M. Beckmann, Assignment problems and the location of economic activities, Econometrica 25 (1957) 23–76.

[25] E. S. Mills, Markets and efficient resource allocation in urban areas, Swedish Journal of Economics 74 (1972) 100–113.

[26] A. L. Zielinska, R. Church, P. Jankowski, Sustainable urban land use allocation with spatial optimization, in: 8th ICA Workshop on Generalisation and Multiple Representation, 2005.

[27] D. P. Ward, A. T. Murray, S. R. Phinn, An optimized cellular automata approach for sustainable urban development in rapidly urbanizing regions, in: Proceedings of the 4th International Conference on GeoComputation, 1999.

[28] D. P. Ward, A. T. Murray, S. R. Phinn, Integrating spatial optimization and cellular automata for evaluating urban change, The Annals of Regional Science 37 (2003) 131–148.

[29] J. Malczewski, GIS-based land-use suitability analysis: A critical review, Progress in Planning 62 (2004) 3–65.

[30] J. Malczewski (Ed.), GIS and Multicriteria Decision Analysis, Wiley, New York, 1999.

[31] J. Aerts, Spatial decision support for resource allocation: Integration of optimization, uncertainty analysis and visualization techniques, Ph.D. thesis, Faculty of Science, University of Amsterdam (2002).

[32] J. T. Diamond, J. R. Wright, Design of an integrated spatial information system for multiobjective land use planning, Environment and Planning B 15 (2) (1988) 205–214.

[33] P. Sinha, Application of linear programming to land suitability analysis, Master's thesis, School of Urban and Public Affairs, University of Texas at Arlington (2010).

[34] R. Krzanowski (Ed.), Spatial Evolutionary Modeling, Oxford Press, Oxford, 2001.

[35] D. Z. Sui, Integrating neural networks with GIS for spatial decision making, Operational Geographer 11 (2) (1993) 13–20.

[36] M. Batty, Y. Xie, Z. Sun, From cells to cities, Environment and Planning B 21 (1994) 31–48.

[37] F. Wang, Improving remote sensing image analysis through fuzzy information representation, Photogrammetric engineering and remote sensing 56 (8) (1990) 1163–1168.

[38] R. Banai, Fuzziness in geographic information systems: Contributions from the analytic hierarchy process, International Journal of Geographical Information Systems 7 (1993) 315–329.

[39] H. Jiang, J. R. Eastman, Application of fuzzy measures in multi-criteria evaluation in GIS, International Journal of Geographic Information Science 14 (2000) 173–184.

[40] J. Zhou, D. L. Civco, Using genetic learning neural networks for spatial decision making in GIS, Photogrammetric Engineering and Remote Sensing 11 (1996) 1287–1295.

[41] R. H. Gimblett, G. L. Ball, A. W. Guise, Autonomous rule generation and assessment for complex spatial modeling, Landscape and Urban Planning 30 (1994) 13–26.

[42] C. J. Brookes, A parameterized region-growing programme for site allocation on raster suitability maps, International Journal of Geographic Information Science 11 (1997) 375–396.

[43] D. Fotakis, E. Sidiropoulos, A new multi-objective self-organizing optimization algorithm (MOSOA) for spatial optimization problem, Applied Mathematics and Computation 218 (9) (2012) 5168–5180.

[44] A. Holzkamper, R. Seppelt, A generic tool for optimising land-use patterns and landscape structures, Environmental Modeling & Software 22 (2007) 1801–1804.

[45] K. B. Matthews, A. R. Sibbald, S. Craw, Implementation of a spatial decision support system for rural land use planning: Intergrating geographic information system and environmental models with search and optimisation algorithms, Computers and Electronics in Agriculture 23 (1999) 9–26.

[46] K. B. Matthews, K. Buchan, A. R. Sibbald, S. Craw, Combining deliberative and computer-based methods for multi-objective land-use planning, Agricultural Systems 87 (2006) 18–37.

[47] S. M. Manson, Agent based dynamic spatial simulation of land use/cover change in the Yucatan Peninsula, in: Fourth International Conference on Integrating GIS and Environmental Modeling, Banff, Canada.

[48] N. Xiao, D. A. Bennett, M. P. Armstrong, Using evolutionary algorithms to generate alternatives for multiobjective site-search problems, Environment and Planning A 34 (4) (2002) 639–656.

[49] S. A. Gabriel, J. A. Faria, G. E. Moglen, A multiobjective optimization approach to smart growth in land development, Socio-economic Planning Sciences 40 (2006) 212–248.

[50] H. H. Zhang, L. Bian, Simulating multi-objective spatial optimization allocation of land use based on the integration of multi-agent system and genetic algorithm, International Journal of Environmental Research 4 (4) (2010) 765–776.

[51] J. Bos, Zoning in forest management: A quadratic assignment problem solved by simulated annealing, Journal of Environmental Management 37 (1991) 127–145.

[52] I. S. Riveira, R. C. Maseda, D. M. Barros, GIS-based planning support system for rural land-use allocation, Computers and Electronics in Agriculture 63 (2008) 257–273.

[53] L. Xiaoli, Y. Chen, L. Daoliang, A spatial decision support system for land use structure optimization, WSEAS Transactions on Computers 8 (3).

[54] A. Anas, D. Pines, Anti-sprawl policies in a system of congested cities, Regional Science and urban Economics 38 (5) (2008) 408–423.

[55] F. Jiang, S. Liu, H. Yuan, Q. Zhang, Measuring urban sprawl in beijing with geo-spatial indices, Journal of Geographical Sciences 17 (4) (2007) 469–478.

[56] X. Hongjie, Toward a compact settlement: A sustainable development way of settlements for chinese city, in: International Conference on Management and Service Science, 2009.

[57] B. Saikia, Urban sprawl and its periphery- a case study of guwahati city and its periphery in Assam (in north-east India), in: Society of Interdisciplinary Business Research (SIBR) 2011 Conference on Interdisciplinary Business Research, 2011.

[58] S. Z. Shahraki, D. Sauri, P. Serra, S. Modugno, F. Seifolddini, A. Pourahmad, Urban sprawl pattern and land-use change detection in Yazd, Iran, Habitat International 35 (4) (2011) 521–528.

[59] K. Muller, C. Steinmeir, M. Kuchler, Urban growth along motorways in Switzerland, Landscape and Urban Planning 98 (1) (2010) 3–12.

[60] X. Tong, X. Zhang, M. Liu, Detection of urban sprawl using a genetic algorithm-evolved artificial neural network classification in remote sensing: A case study in Jiading and Putuo districts of Shanghai, China, International Journal of Remote Sensing 31 (6) (2010) 1485–1504.

[61] X. J. Yu, C. N. Ng, Spatial and temporal dynamics of urban sprawl and two urban-rural transects: A case study of Guangzhou, China, Landscape and Urban Planning 79 (1) (2007) 96–109.

[62] A. Q. Thompson, L. S. Prokopy, Tracking urban sprawl: Using spatial data to inform farmland preservation policy, Land Use Policy 22 (2009) 194–202.

[63] A. Anas, H. J. Rhee, Curbing excess sprawl with congestion tolls and urban boundaries, Regional Science and Urban Economics 36 (4) (2006) 510–541.

[64] H. S. Banzhaf, N. Lavery, Can the land tax help curb urban sprawl? Evidence from growth patterns in Pennsylvania, Journal of Urban Economics 67 (2) (2010) 169–179.

[65] S. Habibi, N. Asadi, Causes, results and methods of controlling urban sprawl, Procedia Engineering 21 (2011) 133–141.

[66] T. J. Stewart, R. Janssen, M. V. Herwijnen, A genetic algorithm approach to multiobjective land use planning, Computers and Operations Research 31 (2004) 2293–2313.

[67] A. Ligmann-Zielinska, R. Church, Spatial optimization as a generative technique for sustainable multi-objective land-use allocation, International Journal of Geographical Information Science 22 (6) (2008) 601–622.

[68] G. Galster, R. Hanson, M. R. Ratcliffe, H. Wolman, S. Coleman, J. Freihage, Wrestling sprawl to the ground: Defining and measuring an elusive concept, Housing Policy Debate 12 (4) (2001) 681–717.

[69] S. Malpezzi, Estimates of the measurement and determinants of urban sprawl in U.S. metropolitan area, Tech. Rep. 99–06, University of Wisconsin Center for Urban Land Economic Research (1999).

[70] G. B. Dantzig, P. Wolfe, Decomposition principle for linear programs, Operations Research 8 (1) (1960) 101–111.

[71] G. B. Dantzig, P. Wolfe, The decomposition algorithm for linear programs, Econometrica 29 (4) (1961) 767–778.

[72] I. Nowak (Ed.), Relaxation and Decomposition Methods for Mixed integer nonlinear programming, Birkhauser Verlag, 2005.

[73] A. M. Geoffrion, G. W. Graves, Multicommodity distribution system design by benders decomposition, Management Science 20 (1974) 822–844.

[74] J.-F. Cordeau, F. Soumis, J. Desrosiers, A benders decomposition approach for the locomotive and car assignment problem, Transportation Science 34 (2000) 133–149.

[75] J. Cordeau, F. Soumis, J. Desrosiers, Simultaneous assignment of locomotives and cars to passenger trains, Operations Research 49 (2001) 531–548.

[76] M. Florian, G. Guerin, G. Bushel, The engine scheduling problem on a railway track, INFOR 14 (1976) 121–128.

[77] E. M. Loiola, N. M. M. de Abreu, P. O. Boaventura-Netto, P. Hahn, T. Querido, A survey for the quadratic assignment problem, European Journal of Operational Research 176 (2) (2007) 657–690.

[78] I. Nadasdi, Surface de potentiel de la population de la province de liège. exemple dapplication de la transformation des champs gographiques discrets en champs continus., Bulletin de la Société Géographique de Liège 7 (1971) 51–60.

[79] K. Kockelman, A. Anjomani, B. M. Paul, D. Nostikasai, A. Tayyebi, G. Kharel, Design and application of accessible land-use modeling tools for texas regions, Tech. rep., Texas Department of Transportation, Austin, TX (2010).